COMPLEX ANALYSIS / 6

2018

① Let $f(z)$ be analytic in Ω. Show that if

a) $|f(z)|$, b) $\arg(f(z))$, c) $\mathrm{Re}\{f(z)\}$

is constant, so is $f(z)$.

② Prove <u>Pick</u>'s lemma: If $f(z)$ is analytic and $|f(z)| < 1$ when $|z| < 1$, then

$$|f'(z)| \leq \frac{1-|f(z)|^2}{1-|z|^2}, \quad |z| < 1.$$

③ Let $\mathbb{D} = \{z \mid |z| < 1\}$. If $f : \mathbb{D} \longrightarrow \mathbb{D}$ is analytic with <u>two</u> fixed points, say

$$f(z_1) = z_1, \quad f(z_2) = z_2, \qquad (|z_k| < 1)$$

then $f(z) \equiv z$.

④ How many roots does $z^7 + z^5 - 8z^3 + 2z + 1$ have in the annulus $1 < |z| < 2$?

⑤ Prove <u>Weierstraß</u>'s theorem: Let $f_k : \Omega \longrightarrow \mathbb{C}$ be analytic, $k = 1, 2, 3, \ldots$ If $f_k \longrightarrow f$ locally uniformly in Ω, then the limit function f is analytic in Ω.