COMPLEX ANALYSIS 23.I.2018

① What Möbius transformation corresponds to a rotation of the z-plane through $90°$ about the point $z = 2$?

② Find the Möbius transformation which carries the points $z = -1, 1, i$ into the points $w = 0, 3, \infty$.

③ Map the domain bounded by the circles $|z| = 1$ and $|z - \frac{1}{4}| = \frac{1}{4}$ conformally onto an annulus whose outer boundary is the circle $|w| = 1$. How large is the radius of the inner circle?

④ Verify that Green's formula can be written as

$$\frac{1}{2i} \oint_{\partial\Omega} f \, dz = \iint_{\Omega} \frac{\partial f}{\partial \bar{z}} \, dx \, dy$$

under suitable assumptions. Here $f = u + iv$ and

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right)$$

$$\oint_{\partial\Omega} P\,dx + Q\,dy = \iint_{\Omega} \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dx\,dy$$

⑤ Radii of convergence for

$$\sum z^{n!}, \quad \sum n^{2018} z^n, \quad \sum n^n z^n \quad ?$$