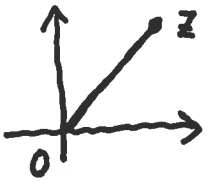
COMPLEX ANALYSIS 3

30.I.2018

① Let $$f(z) = \int_0^z \overline{\zeta}\, d\zeta,$$



where the integration is along the segment from 0 to z. Is $f(z)$ analytic?

② Radius of convergence = ?

$$\sum_{k=0}^{\infty} \left(2 + (-1)^k\right)^k z^k.$$

③ $$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

$$? \quad = \sum_{n=0}^{\infty} n^3 a_n z^n$$

④ Map the domain between $|z| = 1$ and $|z - \frac{1}{2}| = \frac{1}{2}$ on a half plane.

⑤ Prove "Cantor's lemma": If

$$K_1 \supset K_2 \supset K_3 \supset \cdots$$

is a decreasing sequence of nonempty compact sets, then their intersection

$$\bigcap_{n=1}^{\infty} K_n$$

is nonempty. —Show by example that this need not be true if the sets are merely closed.