

COMPLEX ANALYSIS/4

2018

① e^z has the period $2\pi i$. What is the period of $|e^z|$?

② Is $e^{2z} + e^z$ periodic?

③ Map the domain $|z| > 1$ conformally onto the complement of the segment $-1 \leq x \leq 1$. (Joukowski)

④ Suppose, for simplicity, that $h(t)$ is real-valued and continuous, when $0 \leq t \leq 1$.

Prove that

$$H(z) = \int_0^1 \frac{h(t)}{t-z} dt, \quad z \in \mathbb{C} \setminus [0,1]$$

is a) continuous, b) differentiable. c) Show that $H'(z)$ is continuous.

⑤ Prove that

$$e^{a+b} = e^a e^b$$

using the series $e^z = 1 + z + \frac{z^2}{2!} + \dots$.

⑥ Assume that $\sum_{n=0}^{\infty} a_n$ diverges. Show that the sequence $a_n z^n$ ($n=1, 2, 3, \dots$) is unbounded, if $|z| > 1$.