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COMPLEX ANALYSIS / 9

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① Show

$$(1+q)(1+q^2)(1+q^4)(1+q^8)\cdots\cdots = 1+q+q^2+q^3+\cdots, \quad \text{when } |q|<1$$

② Show that

$$\prod_{n=1}^{\infty}\left(1+\frac{i}{n}\right) \text{ diverges}; \qquad \prod_{n=1}^{\infty}\left|1+\frac{i}{n}\right| \text{ converges}. \qquad (i^2=-1)$$

③ Let $c>0$.

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{a^z}{z}\,dz = \begin{cases}1, & a>1,\\ 0, & 0<a<1.\end{cases}$$

④

$$\frac{\sin(z)}{z} = \cos\left(\frac{z}{2}\right)\cos\left(\frac{z}{4}\right)\cos\left(\frac{z}{8}\right)\cdots$$

⑤ Assume that $\sum |a_n|^2 < \infty$. Then $\prod(1+a_n)$ converges if and only if $\sum a_n$ converges.

⑥ Give an example of

$$\prod(1+b_n) = 0 \quad \& \quad \sum b_n \text{ converges}$$

(All $b_n \neq -1$). Diverges to zero.

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