COMPLEX ANALYSIS / 8

2018

① Let $w \neq 0$. Show that every disk $|z| < \delta$ contains infinitely many points satisfying
$$e^{\frac{1}{z}} = w.$$

② Show that the integral
$$li(x) = \underbrace{PV\int_0^2 \frac{dt}{\log(t)}}_{\approx 1{,}04} + \int_2^x \frac{dt}{\log(t)}$$
"INTEGRAL LOGARITHM"

converges.

③ $$\int_0^\infty \frac{x^\alpha}{(1+x)^2}\,dx = \frac{\pi\alpha}{\sin(\pi\alpha)}, \quad -1 < \alpha < 1$$
Key Hole Contour

④ $$\int_0^\infty \frac{\log(x)}{x^3+1}\,dx = -\frac{2\pi^2}{27}, \qquad \int_0^\infty \frac{dx}{1+x^3} = \frac{2\pi}{3\sqrt{3}}$$

Hint: The indented sector $\varepsilon \le r \le R$, $0 \le \theta \le \frac{2\pi}{3}$.

⑤ $$\int_0^\infty \frac{dx}{1+x^{2n}} = \frac{\pi}{2n\sin(\frac{\pi}{2n})}$$
$n = 1, 2, 3, \ldots$

⑥ $$\int_0^1 \frac{dx}{1-x^2} = +\infty$$
$$\int_1^\infty \frac{dx}{1-x^2} = -\infty$$
$$PV\int_0^\infty \frac{dx}{1-x^2} = ?$$