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COMPLEX ANALYSIS / 13

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① Study the convergence of

$$\sum_{n=0}^{\infty} \frac{\sin(nz)}{2^n}$$

in the complex plane.

② How is the unit disk $|z|<1$ mapped under the conformal mapping (the Koebe function)

$$w = \frac{z}{(1-z)^2} = 1 + z + 2z^2 + 3z^3 + 4z^4 + \cdots$$

③ Consider

$$I(z) = \int_\gamma t^{z-1} e^{-t}\,dt$$

The contour γ

Conclude that

$$\Gamma(z) = \frac{1}{e^{2\pi i z}-1} \int_\gamma t^{z-1} e^{-t}\,dt$$

when $\mathrm{Re}(z) > 0$. Conclude that the formula is valid when z is not an integer.

④ Is the inequality

$$\left| e^z - 1 - z - \frac{z^2}{2} \right| \le e^x - 1 - x - \frac{x^2}{2},$$

where $z = x + iy$, true?

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(5) Find a conformal mapping of the "exterior" $|z| > 1$ onto $\mathbb{C} \setminus [-1,1]$.

(6) The inverse of the Laplace transform

$$F(s) = \int_0^\infty f(x) e^{-xs}\, dx$$

is given by

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) e^{sx}\, ds$$

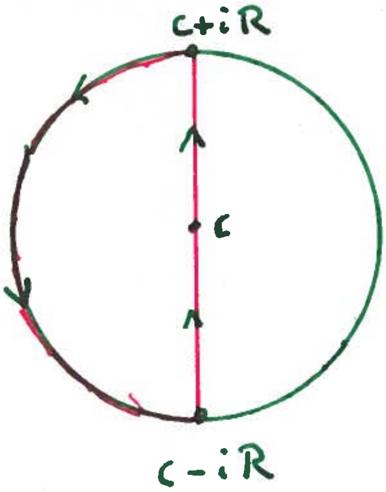
complex!

$z = c + iy$

c = real

where c is so big that the singularities of $F(s)$ are all in the half-plane $\mathrm{Re}(s) < c$.

Use the so-called Bromwich contour



$R \to \infty$

to find the inverse of

$$F(s) = \frac{1}{s(s+1)}$$

$$8i = \infty$$

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