

① $\left| \frac{(3+4i)(-1+2i)}{(-1-i)(3-i)} \right| = ?$

② $\left| \frac{a-b}{1-\bar{a}b} \right| = 1$ if $|a|=1$ or $|b|=1$

(Exception, if $|a|=|b|=1$, $1 = \operatorname{Re}(a\bar{b})$.)

③ Prove $\left| \frac{a-b}{1-\bar{a}b} \right| < 1$
if $|a| < 1$ and $|b| < 1$.

⑦ $P(z) = \sum_{k=0}^n a_k z^k$
Show $\Delta |P(z)|^2 = 4 \left| \frac{\partial P}{\partial \bar{z}} \right|^2$
↑ Laplace operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

④ Prove Lagrange's Identity

$$\left| \sum_{j=1}^n a_j b_j \right|^2 = \sum_{j=1}^n |a_j|^2 \sum_{j=1}^n |b_j|^2 - \sum_{1 \leq i < j \leq n} |a_i \bar{b}_j - a_j \bar{b}_i|^2$$

⑤ When does $az + b\bar{z} + c = 0$ represent a line?

⑥ Prove $\lim z_n = A \implies \lim_{n \rightarrow \infty} \frac{z_1 + z_2 + \dots + z_n}{n} = A$