# Conformal mappings

1. Find the general form of linear transformation which maps the upper half-plane into:

a. Itself  b. Right half-plane.

2. Find the linear function $w = w(z)$ which maps the strip between the lines $x = a$ and $x = a + h$ onto the strip $0 < \operatorname{Re} w < 1$ with normalization $w(a) = 0$

3. In the problems below find the image of the domain $D$ under the action of the mapping $w = w(z)$

a. $D = \{z : x > 0,\ y > 0\}$, $w(z) = \dfrac{z - i}{z + i}$

b. $D = \{z : 0 < \arg z < \frac{\pi}{4}\}$; $w(z) = \dfrac{z}{z - 1}$

4. Find the general form of linear-fractional function which maps the upper half plane onto

a. Itself  b. Right half-plane.

5. Find the linear-fractional mapping which maps the points $-1$, $i$, $1+i$ respectively into the points

a. $i$, $1$, $1+i$;  b. $\infty$, $i$, $1$

6. Find the symmetrical image with respect to the unit circle of the following curves

a) $|z| = \frac{1}{2}$        b) $y = 2$

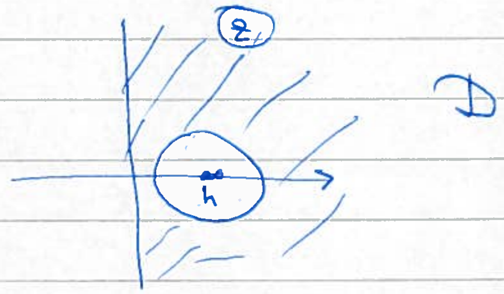
7. Map the disk $|z-2|<1$ onto the disk $|w-2i|<2$ so that $w(2)=i$ ; $\arg w'(2)=0$.

8*. Find $h>1$ such that the domain

$D = \{z : \operatorname{Re} z > 0 \ , \ |z-h| > 1\}$ can be mapped

onto the ring $\{w : 1<|w|<2\}$. Find such

mapping




9. Map the angle $0<\arg z<\pi\alpha$ , $0<\alpha<2$ onto the upper half-plane.

10. Find the function $w(z)$ which maps the semicircle $|z|<1$, $\operatorname{Im} z > 0$ onto the upper half-plane with the conditions
$$w(\pm 1) = \mp 1, \quad w(0) = \infty .$$

11. Map on the upper half-plane the domains

a. $D = \{z : |z| < 2\}$, $0 < \arg z < \alpha\pi$, $0 < \alpha \leq 2$

b. $D = \{z : |z| > 2\}$, $0 < \arg z < \alpha\pi$, $0 < \alpha \leq 2$

c. $D = \{z : \operatorname{Im} z > 0\} \setminus [0, ih]$, $h > 0$.

12. Let $w(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$. Find the images of the domains

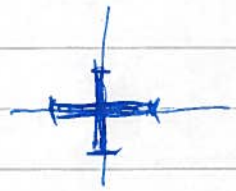
a. $D = \{z : \operatorname{Im} z > 0\}$

b. $D = \{z : |z| < R\}$ where $R < 1$

c. $D = \{z : |z| > R\}$ where $R > 1$

13. Find conformal mapping of the following domains onto the upper half-plane:

a. $\mathbb{C} \setminus ([-a,b] \cup [-ic, ic])$, $a, b, c > 0$



b. $\mathbb{C} \setminus ([-ia, 0] \cup \{z : \operatorname{Im} z < 0,\ |z| = 1\})$, $a > 1$