Exercises for calculating integrals. Residues.

1. Let $\gamma = [0, i]$. Find the integral

$$\int_\gamma z \sin z \, dz$$

2. Calculate the integrals (all circles are oriented contrclockwise)

a. $\displaystyle\int_{|z+i|=3} \sin z \frac{dz}{z+i}$; b. $\displaystyle\int_{|z|=2} \frac{e^z}{z^2-1} dz$;

c. $\displaystyle\int_{|z|=4} \frac{\cos z}{z^2-\pi^2} dz$; d. $\displaystyle\int_{|z|=r} \frac{dz}{(z-a)^n (z-b)}$,

assuming $|a| < r < |b|$, $n = 1, 2, \ldots$

3. Find residues:

a. $\displaystyle\mathrm{Res}_{\frac{\pi}{4}} \frac{\cos z}{z - \pi/4}$ b. $\displaystyle\mathrm{Res}_{\infty} \frac{\sin z}{z^2}$ c. $\displaystyle\mathrm{Res}_{1} \frac{e^z}{(z-1)^2}$

d. $\mathrm{Res}_1 \, z e^{\frac{i}{z-1}}$

4. Find the residus of functions below at all their ~~...~~ finite singular points

a. $\dfrac{1}{z+z^3}$ b. $\dfrac{1}{(z^2+1)(z-1)^2}$ c. $\dfrac{1}{\sin \pi z}$

d. $\operatorname{Cotan} \pi z$ e. $\tan \pi z$ f. $\dfrac{1}{\sin z^2}$

5. Find the integrals

a. $\displaystyle\int_{\partial D} \frac{dz}{(z-1)^2(z+1)}$; $D = \{z : |z-1-i| < 2\}$
∂D is boundary of D.

b. $\displaystyle\int_{\partial D} \frac{z}{z+3} e^{\frac{1}{3z}} dz$, $D = \{z : |z| > 4\}$

c. $\displaystyle\int_{\partial D} \sin \frac{z}{z+1} dz$, $D = \{z : |z| > 3\}$

d. $\displaystyle\int_{\partial D} \frac{z^3 dz}{e^{z^2} - 1} dz$, $D = \{z : |z| < 4\}$.

6. Find the integrals

a. $\displaystyle\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+9)}$ b. $\displaystyle\int_{-\infty}^{\infty} \frac{x^4+1}{x^6+1}\,dx$

c. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$ d. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(a+bx^2)^n}$,

$a, b > 0$, $n = 1, 2, \ldots$

7. Find the integrals

a. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{ix}\,dx}{x^2-2ix-2}$ b. $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+2ix-2)^2}\,dx$

c. $\displaystyle\int_{-i\infty}^{i\infty} \frac{e^{zt}}{(z^2-1)^2}\,dz$, $t > 0$ is a given number.

8. Find the integrals

a. $\displaystyle\int_{-\infty}^{\infty} \frac{(x+1)\sin 2x}{x^2+2x+2}\,dx$ b. $\displaystyle\int_{0}^{\infty} \frac{x\sin x}{x^2+a^2}\,dx$, $a > 0$

9. Find the integrals

a. $\int_0^\infty \frac{dx}{(x+1)\sqrt{x}}$

b. $\int_0^\infty \frac{x^{\alpha-1}}{1+\sqrt[3]{x}}\,dx$, $0<\alpha<1/3$

c. $\int_0^2 \frac{\sqrt{x(2-x)}}{x+3}$

d. $\int_0^\infty \frac{\ln x}{(x+1)\sqrt{x}}\,dx$

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