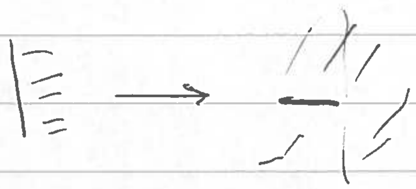
Conformal mappings.

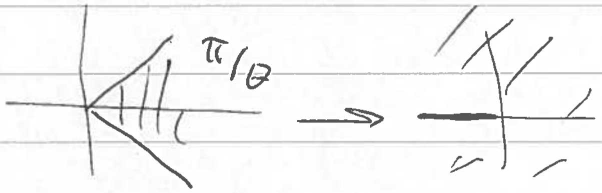
1. Remind the definition.

2. Example $f : \Omega \to \mathbb{C}$, $f'(z) \neq 0$, $z \in \Omega$
is not enough

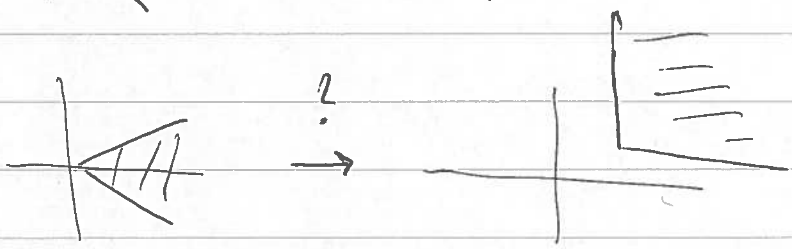
3. Simplest examples



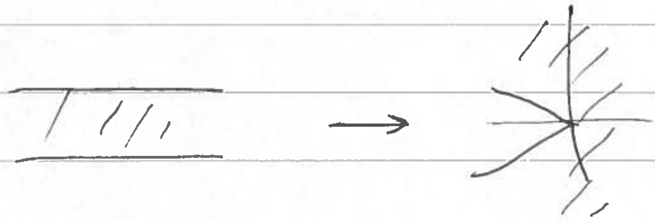
$w = z^2$



$w = z^{\theta}$

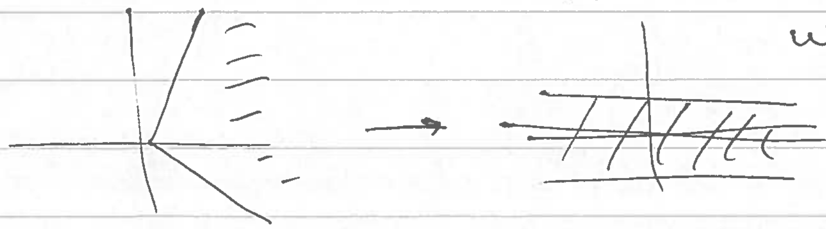


4. Simplest examples (cont).

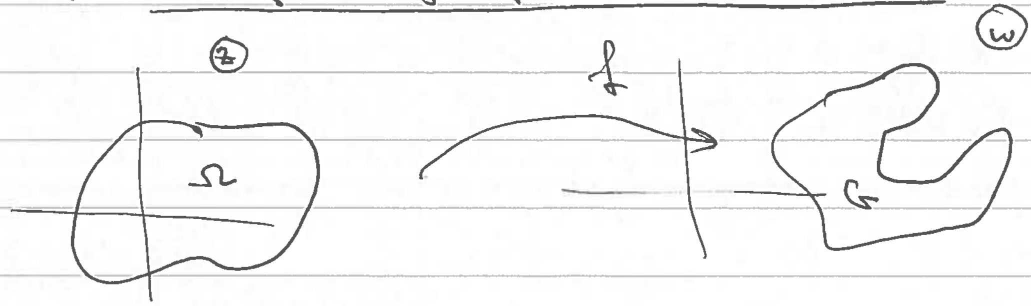


$w = e^{z}$

$w = \ln z$



5. Corresponding of the boundaries.



Ω, G - bounded domains, $\partial\Omega$, ∂G - simple closed curves

$f \in \mathrm{Anal}(\Omega)$, $f : \partial\Omega \to \partial\Omega$ such that $f(z)$ runs once along ∂G in positive direction when z runs along $\partial\Omega$ in positive direction $\Rightarrow$ f is conformal.

Remark: We will see soon that boundedness is extra.

6. Riemann theorem (without proof yet)

Each bounded simply connected domain can be mapped conformally on the unit disk

Remark: Again we will see that boundedness is extra, it suffices to demand that $\partial\Omega$ contains at least two points in $\overline{\mathbb{C}}$.

1. Fractional - Linear transformations

- Definition

- Examples Translation, Affine, Inversion

- This is a mapping of $\overline{\mathbb{C}}$ onto itself

- Inverse and composition of f.-l transformation is also such transformation

- Matrix of a f-l transformation

- Number of free parameters

- Linear-fractional transformation is defined by its values at three points.

- Given z_1, z_2, z_3, w_1, w_2, w_3 there is a unique transformation which maps z_j to w_j

- Any linear-fractional transformation is a combination of dilation, translation and inversion.

- Circles in extended plane:

  straight lines or circles.

  (generalized circles)

- Generalized circles become generalized circles after linear-fractional transformation.

  Consider three cases.

- Images of vertical lines under mappings $w = 1/z$.

  Geometric viewpoint.