Lagrange interpolation formula.

$P(z)$ - polynomial of degree n;
$z_1, \dots, z_{n+1}$ - some points in $\mathbb{C}$,
$z_k \neq z_\ell$ for $k \neq \ell$.

Then:

$$P(z) = \sum_{j=1}^{n+1} P(z_j) \frac{Q(z)}{Q'(z_j)(z-z_j)} \quad (*)$$

where $Q(z) = (z-z_1)(z-z_2)\cdots(z-z_{n+1})$.

Proof. Fix $z \in \mathbb{C}$, then for any R such that $|z| < R$, $|z_j| < R$, $j=1,\dots,n+1$

$$\left.\begin{aligned} &\frac{1}{2i\pi} \int_{|\zeta|=R} \frac{P(\zeta)}{(\zeta - z) Q(\zeta)} d\zeta = \\ (!) \quad &\underbrace{\frac{P(z)}{Q(z)}}_{\text{Residues at } \zeta = z} + \sum_{j=1}^{n+1} P(z_j) \underbrace{\frac{1}{(z_j - z) Q'(z_j)}}_{\text{Residues at } \zeta = z_j} \end{aligned}\right\} \text{By the residues theorem}$$

The right hand side is independent of R!

Further $\deg P(z) = n \Rightarrow |P(z)| \sim |z|^n$ for large $|z|$

$\deg\left((\zeta - z)\, Q(\zeta)\right) = n+2 \Rightarrow |\zeta|^{n+2}$ for large $|\zeta|$

Therefore $\left|\dfrac{P(\zeta)}{(\zeta - z)\, Q(\zeta)}\right| \sim \dfrac{1}{R^2}$ for $|\zeta| = R$, R - large.

That is why

$$\left|\frac{1}{2i\pi} \int_{|\zeta|=R} \frac{P(\zeta)}{(\zeta - z)\, Q(\zeta)}\, d\zeta\right| < \text{Const} \cdot R\, \frac{1}{R^2} \to 0$$

as $R \to \infty$.

Passing to the limit as $R \to \infty$ in (!) we obtain

$$0 = \frac{P(z)}{Q(z)} + \sum_{j=1}^{n+1} P(z_j)\, \frac{1}{(z_j - z)\, Q'(z_j)}$$

which is equivalent to $(*)$.