Exercise, 01.02

1. Fix $n \geq 1$, $r > 0$, and $\lambda = \rho e^{i\phi}$. What is the maximum modulus of $z^n + \lambda$ over the disk $\{|z| < r\}$? Where does $z^n + \lambda$ attains its maximum modulus over this disk?

2. a) Determine whose image under mapping the largest disk around the origin whose image under mapping $w = z^2 + z$ is one to one.
b) The same problem for $w = e^z$.

3. Use the maximum principle to prove the fundamental theorem of algebra that each polynomial of degree $n \geq 1$ has a zero by applying the max principle to $1/p(z)$ on a disk of large radius.

4. Assume $f(z)$ be a *bounded* functions in the right half plane and that f extends continuously to the imaginary axis. Let also $|f(iy)| \leq 1$ for all points iy on the imaginary axis. Prove that then $|f(z)| \leq 1$ in the right half-plane. *Hint* For small $\epsilon > 0$ consider the function $(1 + z)^{-\epsilon} f(z)$ in a large asemidisk.