

Project 2, Complex Analysis  
due Monday March 31, 2014

Problem 1:

$$\text{Let } f(z) = \frac{(\cos z - 1)e^{\frac{-i}{z}}}{z^3(3z^2 - 27)}.$$

- Find the singularities of  $f$  and characterize them.
- Find the integral  $\oint_{|z|=5} f(z) dz$

Problem 2:

Let  $h(z)$  be analytic in  $\Delta = \{z \in \mathbb{C}; |z| < 1\}$ . Suppose that  $h(0) = h'(0) = h''(0) = 0$ . Assume that  $|h| < 1$ .

- Show that  $|h(z)| \leq |z|^3$ .
- Show that if  $h(1/2) = 1/8$ , then  $h\left(\left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)\frac{1}{8}\right) = e^{3i\pi/4}/2^9$ .

Problem 3:

- Show that  $10z^5 + e^z + \cos z$  has exactly 5 zeroes in the unit disc  $\Delta = \{|z| < 1\}$ .
- Show that

$$\frac{1}{2\pi i} \oint_{|z|=1} \frac{50z^4 + e^z - \sin z}{10z^5 + e^z + \cos z} dz = 5.$$

Problem 4:

Find

$$\int_0^\infty \frac{x^{3/2}}{(1+x)^4} dx.$$

Show all estimates.

Problem 5:

- Find the radius of convergence of  $\sum_{n=1}^\infty \left(\frac{i(n^2+3)}{3(2n^2-1)}\right)^n z^n$ .
- Find out where the function

$$g(z) = \sum_{n=1}^\infty \left(\frac{i(n^2+3)}{3(2n^2-1)}\right)^n \left(z^n + \frac{1}{z^n}\right)$$

is analytic.