TMA4170 FOURIER ANALYSIS, WEEK 11, 2018:

(1) In what follows, we assume that $n \geq 1$. Identify a positive trigonometric polynomial $M(\xi)$ satisfying
$$M(\xi) + M(\xi + 2\pi) = 1$$
for all ξ by expanding the right-hand side of the identity
$$1 = (\cos^2(\xi/2) + \sin^2(\xi/2))^{2n+1}$$
by means of the binomial formula.

(2) Use the result of the preceding problem to find a trigonometric polynomial $m_0(\xi)$ such that
$$|m_0(\xi)|^2 = M(\xi).$$

(3) Prove that the solution $m_0(\xi)$ of the preceding problem gives rise to a compactly supported wavelet generating an ONB and satisfying the vanishing moment condition
$$\int \psi(x) x^\ell \, dx = 0, \quad \ell = 0, \ldots, n.$$