

**TMA4170 FOURIER ANALYSIS, WEEK 11, 2018:**

- (1) In what follows, we assume that  $n \geq 1$ . Identify a positive trigonometric polynomial  $M(\xi)$  satisfying

$$M(\xi) + M(\xi + \pi) = 1$$

for all  $\xi$  by expanding the right-hand side of the identity

$$1 = (\cos^2(\xi/2) + \sin^2(\xi/2))^{2n+1}$$

by means of the binomial formula.

- (2) Use the result of the preceding problem to find a trigonometric polynomial  $m_0(\xi)$  such that

$$|m_0(\xi)|^2 = M(\xi).$$

- (3) Prove that the solution  $m_0(\xi)$  of the preceding problem gives rise to a compactly supported wavelet generating an ONB and satisfying the vanishing moment condition

$$\int \psi(x)x^\ell dx = 0, \quad \ell = 0, \dots, n.$$