NTNU – Trondheim
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **TMA4170 Fourier Analysis**

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Examination date: 16th of May 2015

Examination time (from–to): 09:00–13:00

Permitted examination support material: One yellow A4-sized sheet of paper stamped by the Department of Mathematical Sciences. On this sheet the student may write whatever he wants. Specific basic calculator allowed. No other aids permitted.

Other information:
There are 6 problems.

Language: English

Number of pages: 2

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1 Find the sum of the finite Fourier series

$$S(x) = \cos(x) + \cos(3x) + \cos(5x) + \cdots + \cos(2015x)$$

and determine when $S(x) = 0$. (Here x denotes a real number.) *Hint*: Exponential function.

Problem 2 Find the Fourier transform of the function

$$f(x) = e^{-a|x|}, \quad a > 0.$$

Then, *use the result* to calculate

$$\int_{-\infty}^{\infty} \frac{dy}{(a^2 + y^2)(b^2 + y^2)} = ? \qquad (ab \neq 0)$$

Problem 3 *Define* the Fourier transform $\widehat{T}$ of the distribution

$$T(\phi) = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\phi(x) - \phi(-x)}{x} \, dx.$$

Then, *calculate* the Fourier transform $\widehat{T}(\phi)$. Here ϕ is a test function in the Schwartz class $\mathcal{S}$.

Problem 4 Assume that the so-called scaling numbers p_k are known in the formula

$$\phi(x) = \sum_{k=0}^{3} p_k \phi(2x - k).$$

Describe a (numerical) procedure for how to obtain the unknown scaling function ϕ from the formula. No proof is required.

Problem 5 Let

$$\psi(x) = \frac{\sin(\pi x)}{\pi x}.$$

Show that the functions $\psi_k(x) = \psi(x-k)$, $k = 0, \pm 1, \pm 2, \ldots$ are orthonormal:

$$\int_{-\infty}^{\infty} \psi_k(x)\psi_j(x)\,dx = 0 \quad \text{or} \quad 1.$$

Problem 6 Determine the limit

$$\lim_{n\to\infty} \int_a^b f(x)\Big(\cos(nx+n^3)\Big)^2\,dx,$$

where the function f is continuous in $[a,b]$. (Here f is regarded as known and may appear in the answer.)

Some formulas:

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}, \quad \int_{-\infty}^{\infty} \frac{\sin(x)}{x}\,dx = \pi, \quad \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx$$

Good luck!