



Contact during exam:  
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Fourieranalyse (TMA4170)

2012 June 8

Time: 09:00 – 13:00

**Hjelpemidler:** Godkjent kalkulator HP30S eller Citizen SR-270X. Ett A4-ark stemplet fra Instituttet med valgfri paaskrift av studenten.

1. Let

$$f(x) = \begin{cases} 1, & \text{when } -1 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the Fourier transform of the convolution  $f * f * f$ .

2. Does there exist a function  $f \in L^2(-\pi, \pi)$  whose Fourier coefficients are

$$c_n = \frac{2}{1 + \sqrt{|n|}} \quad ?$$

3. Suppose that  $T$  is a distribution in the class  $\mathcal{D}'(\mathbb{R})$ . Assume that its derivative

$$\frac{dT}{dx} = 0.$$

Prove that  $T$  is the constant distribution.

4. Let  $g \in L^2(\mathbb{R})$  and define

$$\hat{g}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega x} g(x) dx.$$

Assume that

$$2\pi \sum_{k=-\infty}^{\infty} |\hat{g}(\omega + 2k\pi)|^2 \equiv 1.$$

Prove that the functions  $g_k = g(x - k)$ ,  $k = 0, \pm 1, \pm 2, \dots$ , are orthonormal.

5. The scaling function  $\phi(x)$  in a multi-resolution analysis satisfies the scaling relation

$$\phi(x) = \sum_{k=0}^5 p_k \phi(2x - k).$$

Find (an interval containing) the support of  $\phi(x)$ . Find (an interval containing) the support of the corresponding wavelet  $\psi(x)$ .

6. How do you define the Fourier transform  $\widehat{T}$  of a distribution in the Schwartz class  $\mathcal{S}'(\mathbb{R})$ ? Find the Fourier transform of  $h(x) = e^{2ia\pi x}$ , where  $a$  is a real number. (First, define the corresponding distribution.)

7. Prove Poincaré's inequality

$$\int_{-\pi}^{\pi} \left( f(x) - \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt \right)^2 dx \leq \int_{-\pi}^{\pi} f'(x)^2 dx$$

for all  $f \in C^2[-\pi, \pi]$ . For which functions does equality hold?