TMA4170 Fourier Analysis Spring 2017

Exercise set 11 solution

1 **B&N: 6.2** We start by looking at

$$\hat{\psi}(\xi) = -e^{-\frac{i\xi}{2}} P_N\left(-e^{\frac{i\xi}{2}}\right)\hat{\phi}_N\left(\frac{\xi}{2}\right),$$

where

$$P_N(z) = (z+1)^N \tilde{P}_N(z).$$

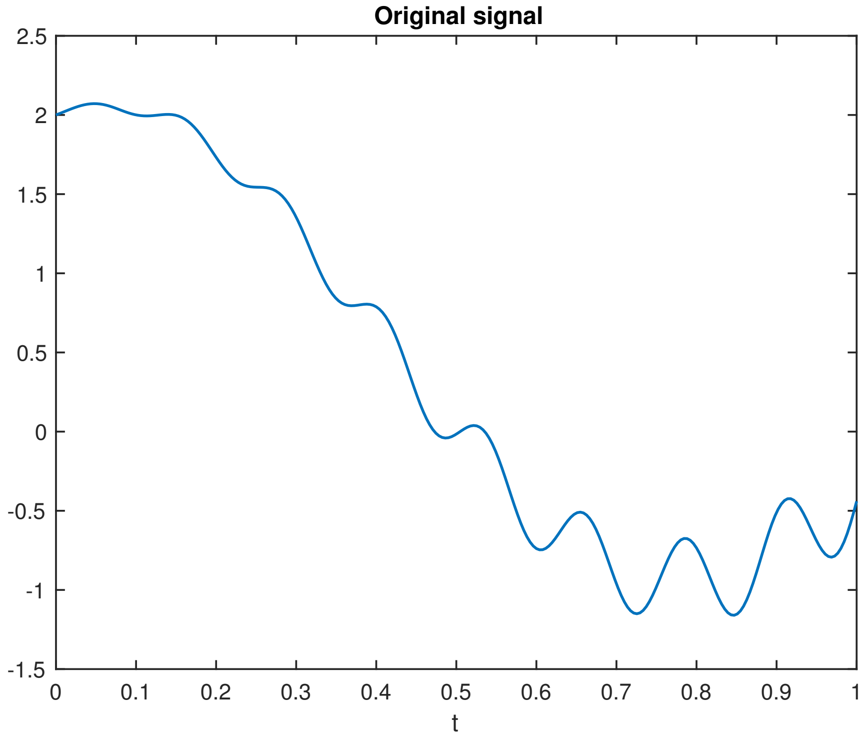
Note that since the root $z=-1$ is an N'th order root of P_N, any derivatives of $\hat{\psi}$ of order less than N will have a root at $\xi = 0$ since the argument of P_N (or derivatives thereof) will be -1. Also, considering the N'th order derivative, this means all terms except the one containing $P_N^{(N)}$ will be zero, such that (after use of the chain rule)

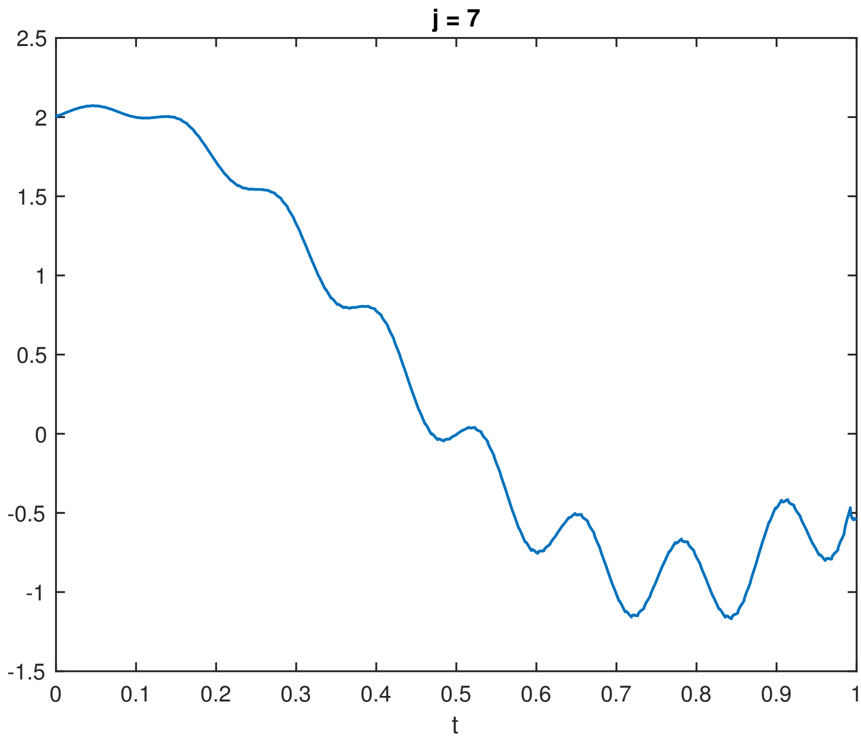
$$\hat{\psi}(0) = -\left(-\frac{i}{2}\right)^N P_N^{(N)}(-1)\,\hat{\phi}_N(0)\,.$$

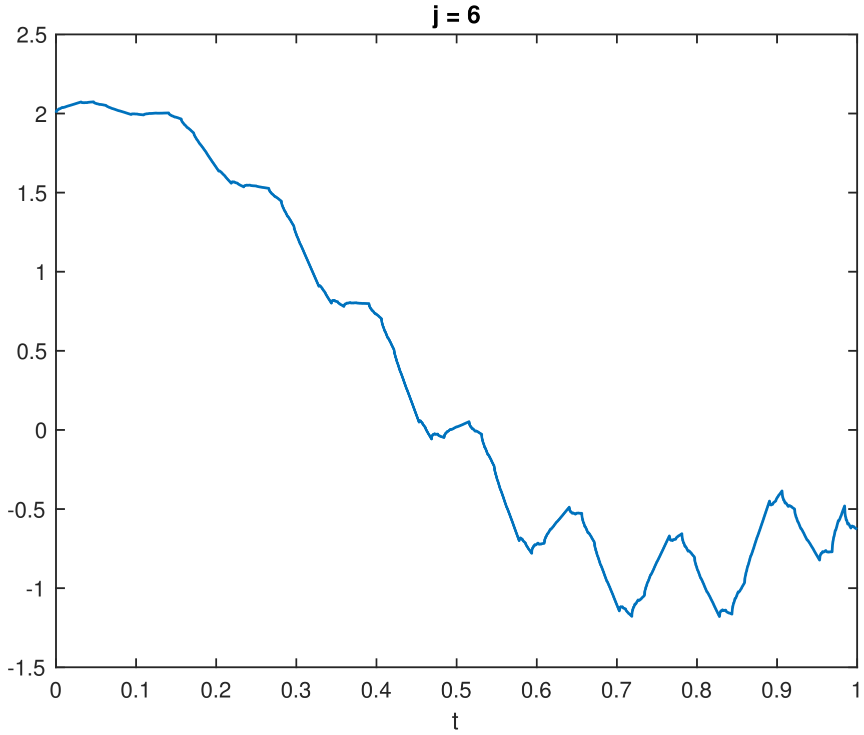
It is easy to see that $P_N^{(N)}(-1) = N!\tilde{P}_N(-1)$, and since $\hat{\phi}(0) = 1/\sqrt{2\pi}$, the result follows.

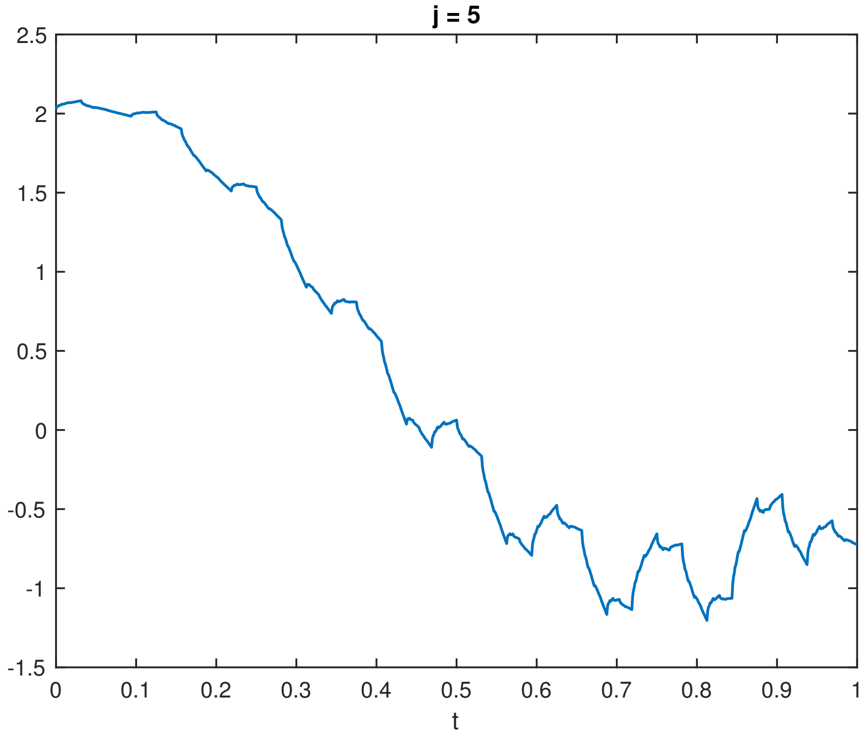
2 **B&N: 6.4**

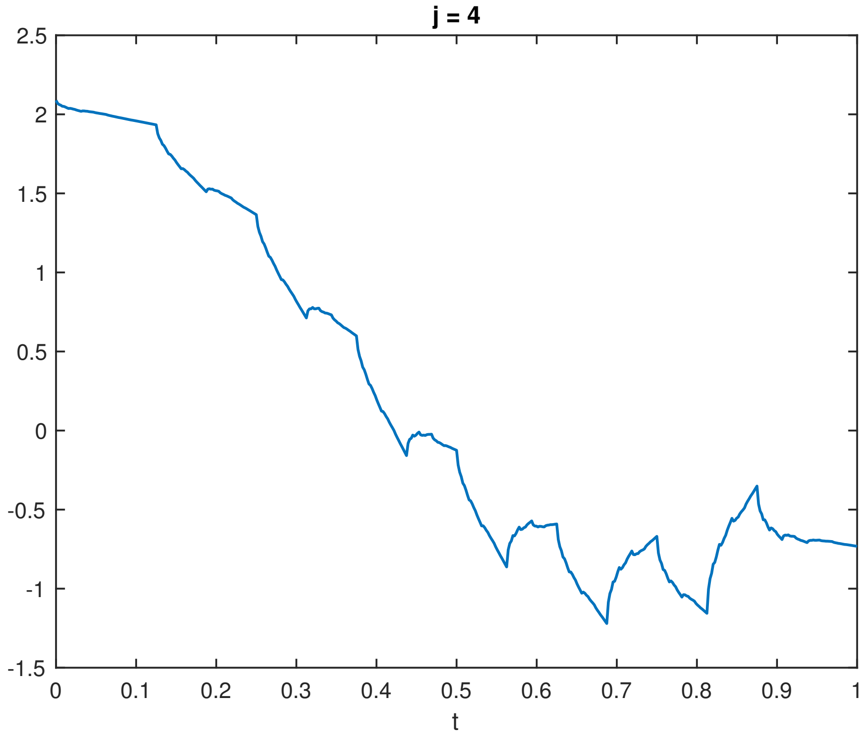
Exercise 4.9: Below are figures showing the components f_j for $j = 1,\ldots,7$ as well as the original signal.

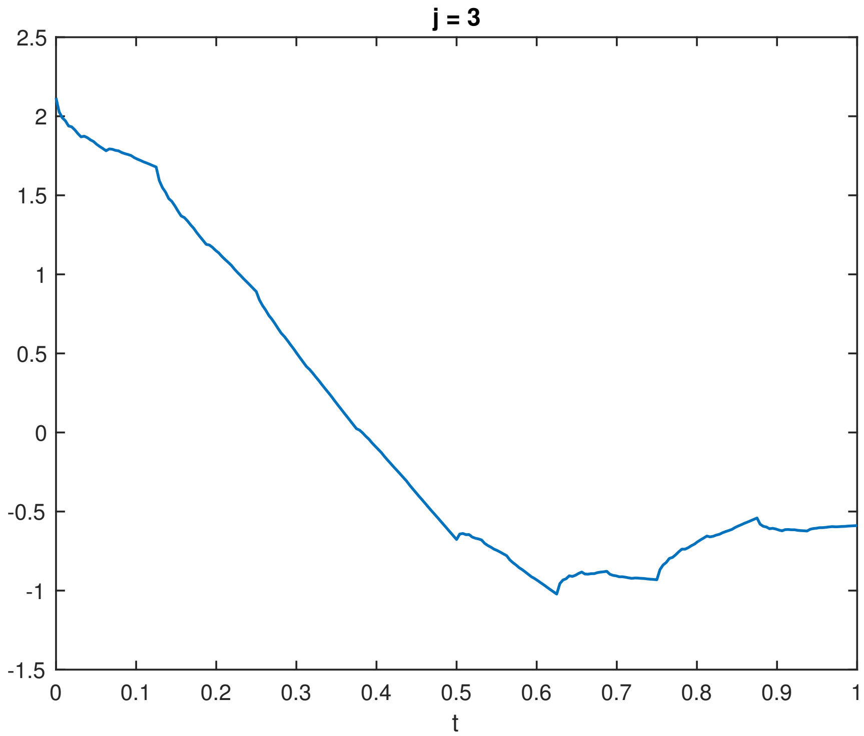

Original signal
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

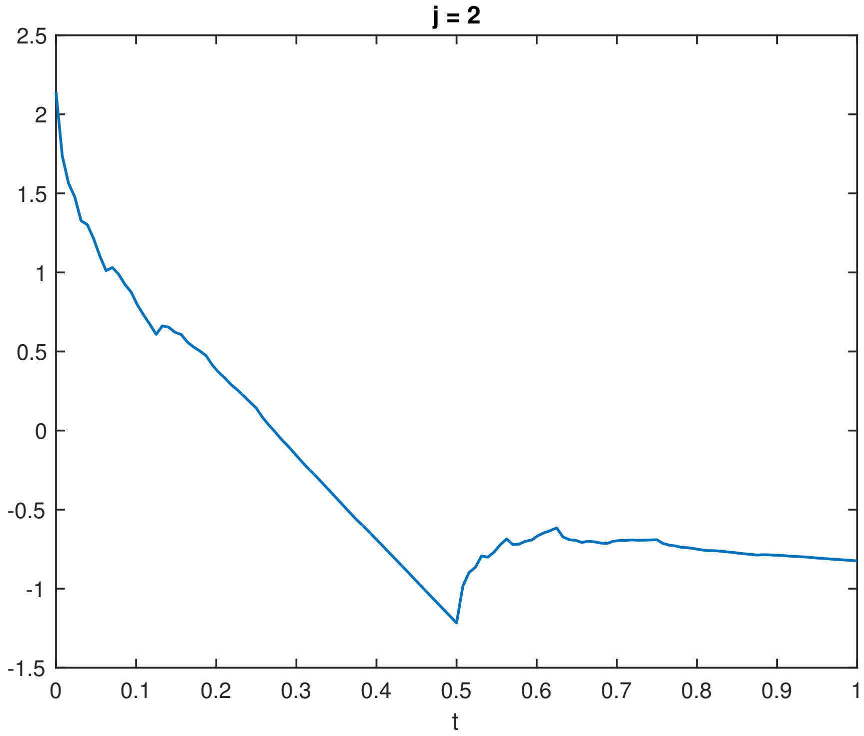

j = 7
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

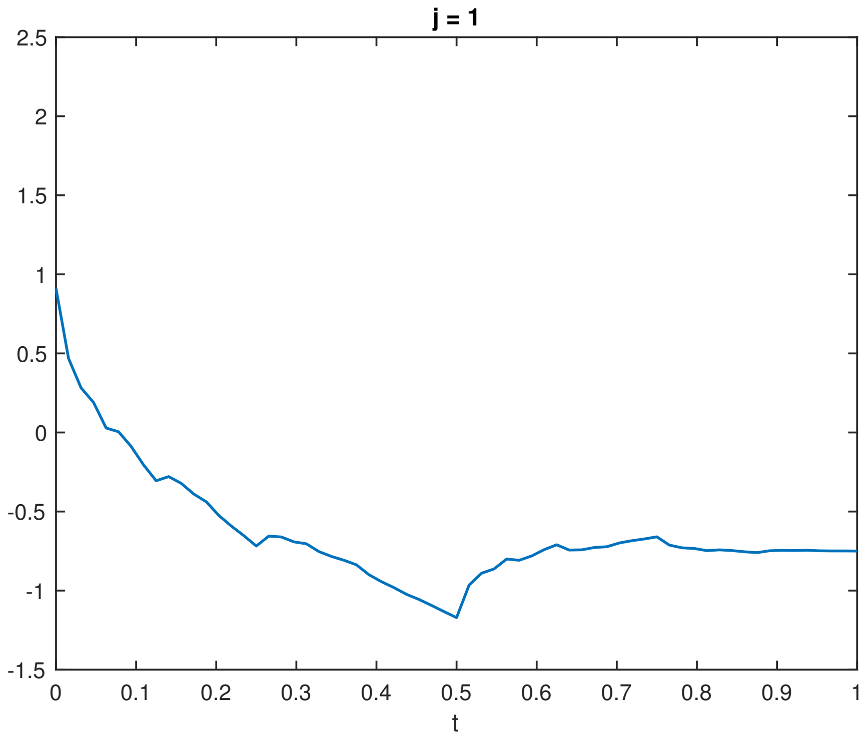

j = 6
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


j = 5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

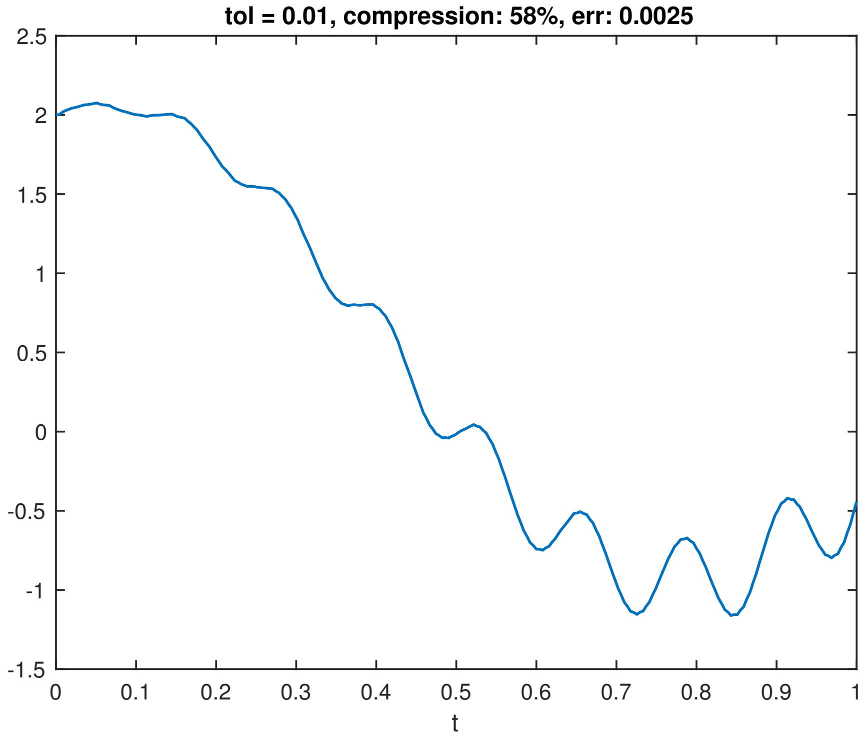

j = 4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

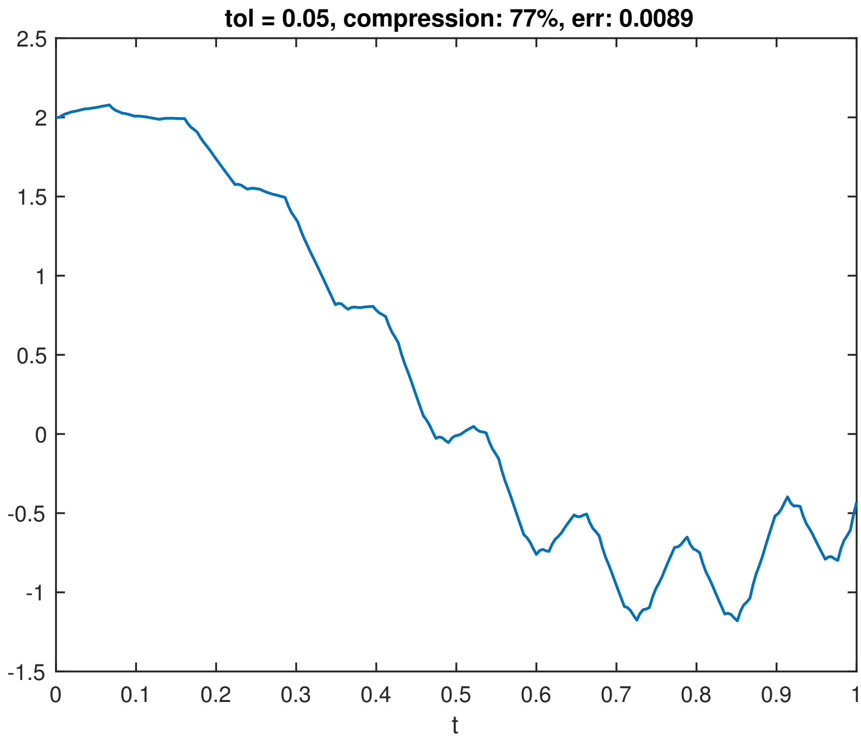

j = 3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

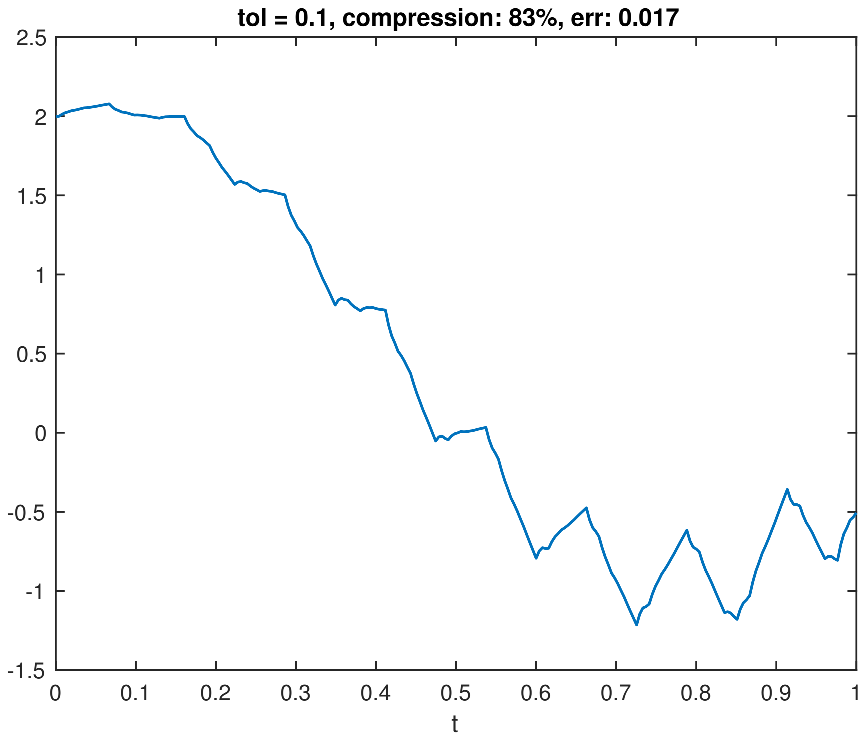

j = 2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

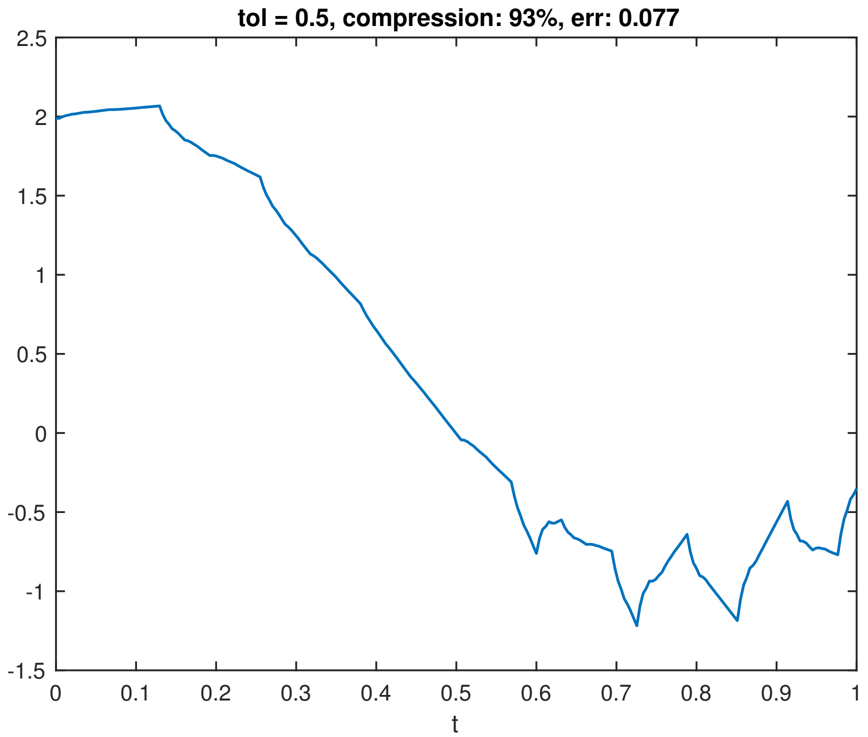

j = 1
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

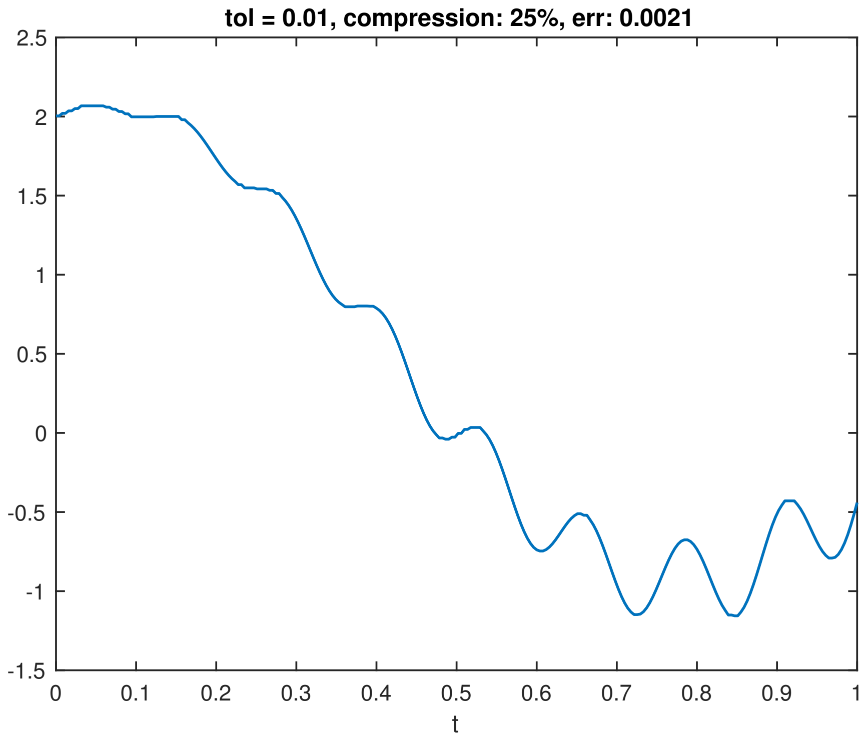
Exercise 4.10: Below are the results with different tolerances, first four plots with Daubechies' wavelets and then four plots with Haar wavelets. Note that the Daubechies' wavelets result in a higher compression rate with lower l^2 error than the Haar wavelets.

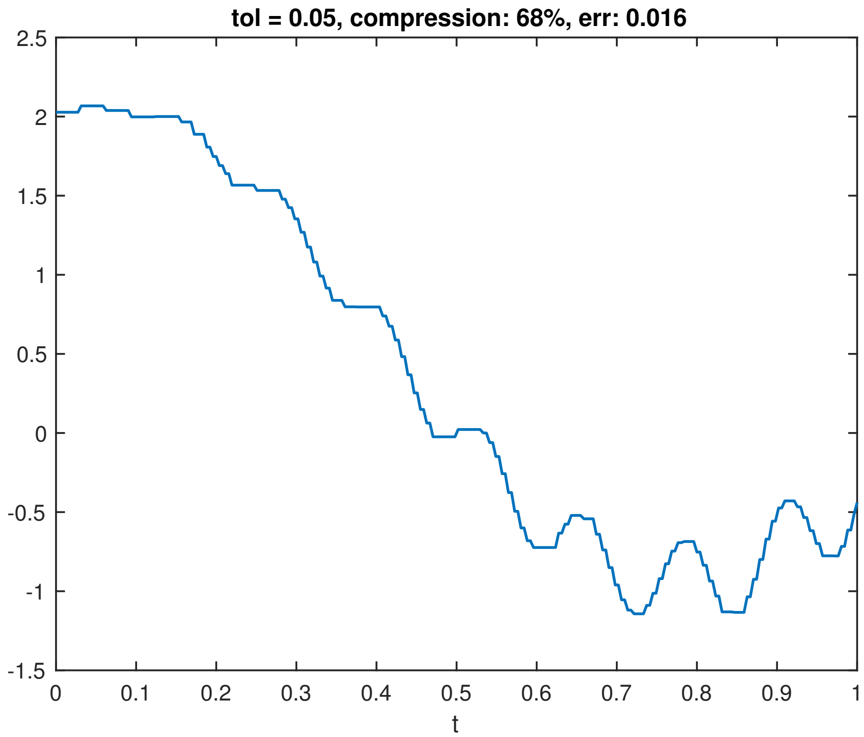


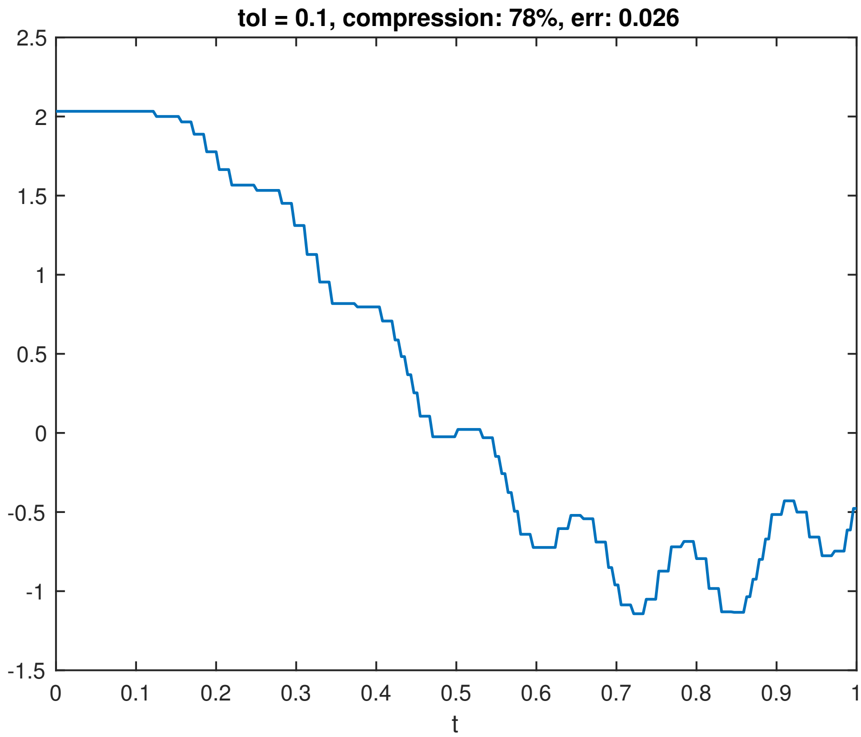


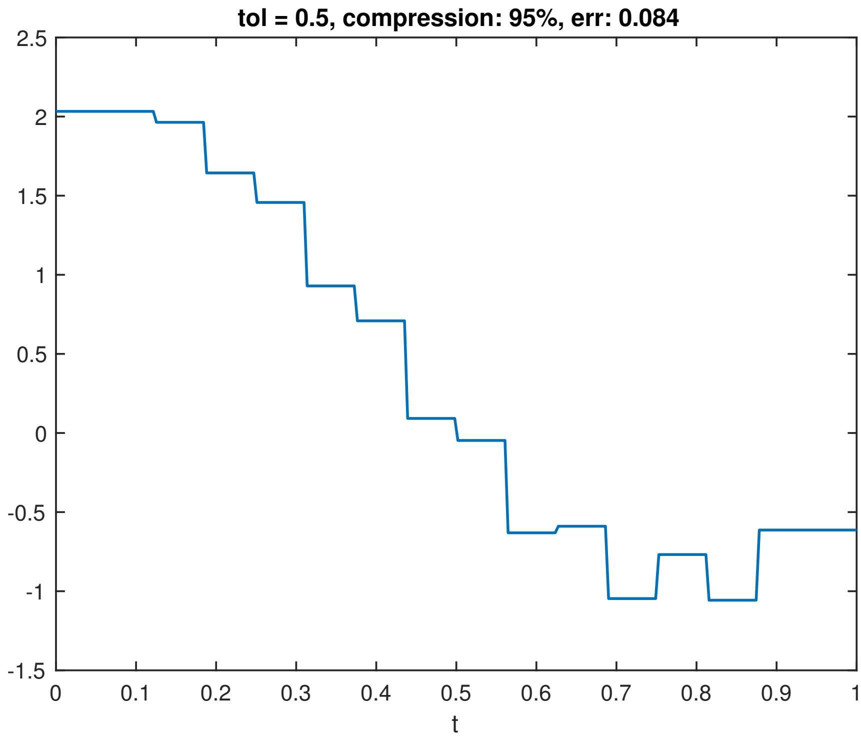

tol = 0.1, compression: 83%, err: 0.017
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


tol = 0.5, compression: 93%, err: 0.077
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


tol = 0.01, compression: 25%, err: 0.0021
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


tol = 0.05, compression: 68%, err: 0.016
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


tol = 0.1, compression: 78%, err: 0.026
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


tol = 0.5, compression: 95%, err: 0.084
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

3 **B&N: 6.6** The following plots show the signal and the highest level detail coefficients. Notice that the detail coefficients are clearly largest (in absolute value) in the indices corresponding to the corner points at $t = 0$ and $t = 1$.