FOURIER ANALYSIS

① Use Plancherel's formula for the L^2-norms to prove that

$$\langle f, g \rangle = \langle \hat{f}, \hat{g} \rangle, \text{ if } f, g \in L^2(\mathbb{R})$$

② Show $$\int_{\pi}^{\infty} \left| \frac{\sin x}{x} \right| dx = +\infty.$$

③ Use a contour integral to obtain

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-i\omega x}\, dx}{\cosh\left(x\sqrt{\frac{\pi}{2}}\right)} = \frac{1}{\cosh\left(\omega\sqrt{\frac{\pi}{2}}\right)}.$$

④ Verify that

$$\iint\limits_{a < x_1 + x_2 < b} \phi(x_1)\phi(x_2)\, dx_1\, dx_2 = \int_a^b (\phi * \phi)(t)\, dt$$

under suitable assumptions. One also has

$$\iint \cdots \int\limits_{a < x_1 + x_2 + \cdots + x_n < b} \phi(x_1)\phi(x_2) \cdots \phi(x_n)\, dx_1\, dx_2 \cdots dx_n$$

$$= \int_a^b (\phi * \phi * \cdots * \phi)(t)\, dt$$

⑥ II.146.

⑤ $$\int_{-\infty}^{\infty} f(x-y) f(y)\, dy = e^{-x^2}, \quad f(x) = ?$$