NTNU – Trondheim
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **TMA4170 Fourier Analysis**

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Examination date: 10th of June 2016

Examination time (from–to): 09:00–13:00

Permitted examination support material: One yellow A4-sized sheet of paper stamped by the Department of Mathematical Sciences. On this sheet the student may write whatever he wants. Specific basic calculator allowed. No other aids permitted.

Other information:
There are 6 problems: 1, 2, 3a, 3b, 4, 5.

Language: English

Number of pages: 2

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1 Use the formula

$$|\sin(x)| = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{(\sin(nx))^2}{4n^2-1}$$

to evaluate the sum

$$\sum_{n=1}^{\infty}\frac{1}{(4n^2-1)^2}.$$

Hint: Watch up!

Problem 2 In three variables we use the Fourier transform

$$\hat{f}(\xi) = \iiint_{\mathbb{R}^3} f(\mathbf{x})e^{-2\pi i\langle \mathbf{x},\xi\rangle}\,dx_1dx_2dx_3.$$

First, show that the Fourier transform of the function

$$\frac{1}{|\mathbf{x}|} = \frac{1}{\sqrt{x_1^2+x_2^2+x_3^2}}$$

depends only on $|\xi| = \sqrt{\xi_1^2+\xi_2^2+\xi_3^2}$. Then calculate it.

Hint. Bring ξ to a suitable position.

Problem 3 Gibbs' Phenomenon. The function

$$f(x) = \begin{cases} 1, & 0 < x < \pi, \\ 0, & -\pi < x < 0, \end{cases}$$

has the Fourier expansion

$$\frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{(2n-1)\pi}\sin((2n-1)x)).$$

Consider the partial sums

$$S_{2N}(x) = \frac{1}{2} + \sum_{n=1}^{N}\frac{2}{(2n-1)\pi}\sin((2n-1)x)).$$

a) Find the sum $\frac{dS_{2N}(x)}{dx}$ of the differentiated series in a convenient form and use it to find the first positive maximum point x_{2N} of $S_{2N}(x)$.

b) Determine

$$G = \lim_{N\to\infty} S_{2N}(x_{2N})$$

and decide which of the alternatives

$$G > 1, \qquad \frac{1}{2} < G \leq 1, \quad \textit{or} \quad G = \frac{1}{2}$$

is true.

Problem 4 *Define* the Fourier transform $\widehat{T}$ of the distribution

$$T(\phi) = -\int_{-\infty}^{\infty} \frac{d\phi(x)}{dx} \log(|x|)\, dx.$$

Then, *calculate* the Fourier transform $\widehat{T}(\phi)$. Here ϕ is a test function in the Schwartz class $\mathcal{S}(\mathbb{R})$.

Hint: First, isolate the origin and integrate by parts.

Problem 5 Let $g \in L^2(\mathbb{R})$. Assume that

$$2\pi \sum_{n=-\infty}^{\infty} |\hat{g}(\omega + 2n\pi)|^2 = 1 + 2cos(\omega).$$

Calculate the numbers

$$\gamma_k = \int_{-\infty}^{\infty} g(x)\overline{g(x-k)}\, dx, \qquad k = 0, \pm 1, \pm 2, \ldots.$$

(The answer must not contain g.)

Some formulas:

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}, \quad \int_{-\infty}^{\infty} \frac{\sin(x)}{x}\, dx = \pi, \quad \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

$$\frac{2}{\pi} \int_0^{\pi} \frac{\sin(x)}{x}\, dx = 1.18...$$

Good luck!