FOURIER ANALYSIS / week 9 2015

① Apply Poisson's Summation Formula

$$\sum_{-\infty}^{\infty} f(2n\pi) = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} \hat{f}(n)$$

to the function $f(x) = e^{-a|x|}$, $a > 0$.

③ Define the distribution x^{-2} as

$$\langle x^{-2}, \phi \rangle = \int_{-\infty}^{\infty} \frac{\phi(x) + \phi(-x) - 2\phi(0)}{2x^2}\, dx$$

when $\phi \in C_0^\infty(\mathbb{R})$. (Thus it is equal to $\int \phi(x)/x^2\, dx$ if it so happens that $\phi(0) = 0$.)

Show that

$$\langle x^{-2}, \phi \rangle = \lim_{\varepsilon \to 0+} \int_{-\infty}^{\infty} \frac{x^2 - \varepsilon^2}{(x^2+\varepsilon^2)^2} \phi(x)\, dx.$$

② Find the Fourier transforms of (the distributions)

$PV\left(\frac{1}{x}\right)$ and $\mathrm{sign}(x)$.

④ The same for x^{-2} above and $|x|$.

⑤ Verify the "Change of Hats" formula

$$\int f(u)\hat{g}(u)\, du = \int \hat{f}(v) g(v)\, dv. \quad (f, g \in L^1(\mathbb{R}))$$