

FOURIER 2015/week 16

① Prove Wirtinger's Inequality

$$\int_{-\pi}^{\pi} |f(x)|^2 dx \leq \int_{-\pi}^{\pi} |f'(x)|^2 dx$$

where $\int_{-\pi}^{\pi} f(x) dx = 0$.

Assume what you need! When do we have equality?

② Assume that $f \in C^N(\mathbb{R})$ has period 2π

Show that

$$|c_k| \leq \frac{\text{const.}}{|k|^N}, \text{ where } c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx.$$

Use $\cosh(x) = \frac{\sinh(\pi)}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{1+n^2} e^{inx} \quad (|x| \leq \pi)$

to find

③ $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ ④ $\sum_{n=1}^{\infty} \frac{1}{(1+n^2)^2}$

⑤ Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Find

$\underbrace{(f * f * \dots * f)}_{n \text{ factors.}}(x)$. What happens when $n \rightarrow \infty$?

Hint: Fourier Transf. of Convolution.