FOURIER ANALYSIS / last

(1) $\cos(x) + \cos(3x) + \cos(5x) + \cdots + \cos((2N-1)x) = ?$

(2) Show that

$$\frac{1}{2} + \cos(x) + \cos(2x) + \cos(3x) + \cdots$$

diverges at <u>every</u> point.

(3) Why is the function

$$\sum_{n=1}^{\infty} \frac{\cos(n^2 x)}{n^2}$$

continuous? <u>Hint</u> Weierstrass's M-test.

(4) $$|\sin(x)| = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin^2(nx)}{4n^2 - 1}$$

(5) $$f(x) = \sum_{\nu} c_\nu e^{i\nu x} \Longrightarrow$$

$$\frac{1}{N} \sum_{k=0}^{N-1} f\left(\frac{x}{N} + \frac{2\pi k}{N}\right) = \sum_{\nu} c_{\nu N} e^{i\nu x}$$

(6) $\widehat{f(\omega)} = ?$; $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}\{f\})) = ?$

(7) $\widehat{\sin(x)} = ?$; $\widehat{[\sin(x)]_+} = ?$

Distributions!

+ = pos. part.

(9) $f(x) \geq 0$

$\Rightarrow |c_0| \leq |c_n|$

for all indices n

Fourier coefficients!

"Fourier's Ring"

(10) The unit circle is totally insulated with initial temperature $f(\theta)$. Solve

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial \theta^2}, \quad t > 0$$

$$u(\theta, 0) = f(\theta)$$