FOURIER ANALYSIS / week 8 2015

① Use a contour integral to prove that

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{-i\omega x}\,dx}{\cosh\left(x\sqrt{\frac{\pi}{2}}\right)} = \frac{1}{\cosh\left(\omega\sqrt{\frac{\pi}{2}}\right)}$$

!

② Verify that

$$\iint_{a<x_1+x_2<b}\phi(x_1)\phi(x_2)\,dx_1\,dx_2 = \int_a^b(\phi*\phi)(t)\,dt,$$

under suitable assumptions. Also

$$\iint\cdots\int_{a<x_1+x_2+\cdots+x_n<b}\phi(x_1)\phi(x_2)\cdots\phi(x_n)\,dx_1\,dx_2\cdots dx_n$$

$$= \int_a^b(\phi*\phi*\cdots*\phi)(t)\,dt.$$

③ $N=4$. Let

$$\hat{y}_k = \sum_{j=0}^{3} y_j\, e^{-\frac{2\pi i}{4}jk} \qquad (k=0,1,2,3)$$

or $\hat{y} = \overline{F_4}\,y$. Construct the matrices

F_4, F_4^{-1} and $\overline{F_4}$.

Calculate $\det(F_4)$. Find the eigenvalues of $\overline{F_4}$.

(4) Let

$$\phi(x) = \frac{\sin(\pi x)}{\pi x}.$$

Prove that the functions

$$\phi_k = \phi_k(x) = \phi(x-k), \quad k = 0, \pm 1, \pm 2, \ldots$$

are orthonormal in $L^2(\mathbb{R})$.

(5) Assume that the function $f \in C_0(\mathbb{R}^2)$ has the property that the integral

$$\int_L f(x,y)\, ds = 0$$

along every line L. (ds = the length measure on L). Is it true that necessarily $f \equiv 0$?