Fourier Analysis / week 7

2015
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① Use Plancherel's formula for the L^2-norms to prove that
$$\langle f, g\rangle = \langle \hat{f}, \hat{g}\rangle, \quad \text{if } f, g \in L^2(\mathbb{R}).$$

② We say that $f(\bar{x})$ is a radial function if $f(A\bar{x}) = f(\bar{x})$ for all orthonormal matrices A, i.e.
$$A^{-1} = A^T, \quad \det(A) = 1.$$
"rotations"

Show that

$\mathbb{R}^n$
$$\hat{f}(\bar{\xi}) = \int_{\mathbb{R}^n} e^{-2\pi i\langle \bar{x}, \bar{\xi}\rangle} f(\bar{x})\, d\vec{x}$$
is a radial function, if $f(\bar{x})$ is.

③ Let $f_0(r) = f(x, y, z)$ for the radial function f, $r = \sqrt{x^2+y^2+z^2}$. Verify

$\mathbb{R}^3$
$$\hat{f}(\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\, e^{-2\pi i(x\xi_1 + y\xi_2 + z\xi_3)}\, dx\, dy\, dz$$
$$= \frac{2}{|\bar{\xi}|}\int_0^{\infty} f_0(r)\sin(2\pi r|\bar{\xi}|)\, dr$$
where $|\bar{\xi}| = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$.

④ Show $\displaystyle\int_0^{\infty}\left|\frac{\sin(x)}{x}\right| dx = \infty.$

⑤ Chapt. 2/13