FOURIER ANALYSIS / week 6

2015

① $\left| e^{i\theta} - 1 \right| \leq |\theta|$, θ = real number.

② $$\int_0^1\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx\,dy \neq \int_0^1\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy\,dx \quad !$$

Calculate the integrals. Also show that

$$\int_0^1\int_0^1 \frac{|x^2-y^2|}{(x^2+y^2)^2}\,dx\,dy = \infty.$$

<u>FUBINI'S THM</u> If $\int_0^1\int_0^1 |f(x,y)|\,dx\,dy < \infty$, then

$$\int_a^b\int_c^d f(x,y)\,dx\,dy = \int_c^d\int_a^b f(x,y)\,dy\,dx.$$

(The integrals are equal also, if $f(x,y) \geq 0$.)

③ $\widehat{e^{-x^2/2}} = e^{-\frac{\omega^2}{2}}$. Find this by differentiating

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-i\omega x} e^{-\frac{x^2}{2}}\,dx.$$

④ Calculate

$$\widehat{e^{-\frac{|x|^2}{2}}} = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} e^{-i\langle \omega, x\rangle} e^{-\frac{|x|^2}{2}}\,dx_1 \ldots dx_n$$

in n dimensions.

⑤ 2/2 ⑥ 2/5 (both from Chapter 2)