

FOURIER ANALYSIS week 11/2015

① Suppose that the scaling numbers p_k are given. Start the iteration

$$\phi_{n+1}(x) = \sum_{k=0}^3 p_k \phi_n(2x-k)$$

with $\phi_1 = \phi_{\text{HAAR}}$. Construct a bounded interval so that

$$\text{supp}(\phi_N) \subset [a, b], \quad N = 1, 2, 3, \dots,$$

for each N .

② Prove that

$$2\pi |\hat{\phi}(\omega)| \leq 1 \quad (\text{for a.e. } \omega \in \mathbb{R})$$

where ϕ is the scaling function.

③ Let, as usual,

$$\phi(x) = \sum_k p_k \phi(2x-k)$$

$$\psi(x) = \sum_k (-1)^k \overline{p_{1-k}} \phi(2x-k)$$

Verify that $\psi(x-n) \perp \phi(x-m)$ for all integers.

④ Prove
$$\frac{\sin \theta}{\theta} = \prod_{n=1}^{\infty} \cos\left(\frac{\theta}{2^n}\right).$$