FUNDAMENTAL SOLUTION OF LAPLACE'S EQN.

$$\begin{cases} \hat{f}(\xi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-2\pi i \langle x, \xi \rangle} f(x)\, dx_1\, dx_2\, dx_3 \\ f(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{+2\pi i \langle x, \xi \rangle} \hat{f}(\xi)\, d\xi_1\, d\xi_2\, d\xi_3 \end{cases}$$

$$\widehat{\frac{\partial u}{\partial x_k}} = -2\pi i\, \xi_k\, \hat{u}(\xi)$$

$$\widehat{\Delta u} = -4\pi^2 |\xi|^2 \hat{u}(\xi) \qquad |\xi|^2 = \xi_1^2 + \xi_2^2 + \xi_3^2$$

$$\Delta u = -c\,\delta$$

$$-4\pi^2 |\xi|^2 \hat{u}(\xi) = -c \cdot 1$$

Has to be calculated!

$$\hat{u}(\xi) = \frac{c}{4\pi^2 |\xi|^2} = \frac{c}{4\pi^2} \widehat{\frac{\pi}{|x|}}$$

$$u(x) = \frac{c}{4\pi |x|} \qquad \boxed{\Delta\left(\frac{1}{r}\right) = -4\pi\,\delta.}$$

Remark: Instead, one can use "Green's 2nd Formula" in the domain $\mathbb{R}^3 \setminus B_\varepsilon(0)$

$$\iiint_{\{r \geq \varepsilon\}} \left(\phi\, \Delta\left(\frac{1}{r}\right) - \frac{1}{r} \Delta \phi\right) d^3x = \oiint_{r=\varepsilon} \left(\phi \frac{\partial}{\partial n}\left(\frac{1}{r}\right) - \frac{1}{r} \frac{\partial}{\partial n}(\phi)\right) dS$$