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Exercise 9 (8.IV.2013)

FOURIER ANALYSE

The multiresolution analysis gives

$$\phi(x) = \sum p_k \phi(2x-k) \qquad \text{SCALING RELATION}$$

$$\psi(x) = \sum (-1)^k \bar{p}_{1-k}\, \phi(2x-k)$$

① Prove $\langle \phi, \psi \rangle = 0$.

② Prove $\langle \psi_{0k}, \psi_{0\ell} \rangle = \delta_{k\ell}$.

③ Suppose that $\phi \in C_0(\mathbb{R})$ {compact support} satisfies the so-called scaling relation above. Show that the endpoints

$$a = \inf\{x \mid \phi(x) \neq 0\}, \quad b = \sup\{\cdots\}$$

are integers.

④ Find the support of the scaling function ϕ in

$$\phi(x) = \sum_{k=0}^{3} p_k \phi(2x-k) \qquad \text{(4 terms)}$$

Then find the support of the wavelet $\psi(x)$. (We mean the largest possible supports.)

(5) Construct a distribution T such that

$$\frac{dT}{dx}(\varphi) = \int_0^\infty \frac{\varphi(x)+\varphi(-x)-2\varphi(0)}{x^2}\,dx$$

<u>Hint</u>: $\dfrac{x}{x^2+\varepsilon^2}$, $\dfrac{x^2-\varepsilon^2}{(x^2+\varepsilon^2)^2}$