FOURIER ANALYSIS / Øv. 7

$$\hat{f}(k) = \sum_{j=0}^{N-1} f(j)\, e^{-\frac{2\pi i}{N} jk}, \qquad f(j+N) \equiv f(j)$$

Prove that

① $\widehat{f * g}(k) = \hat{f}(k)\,\hat{g}(k)$ convolution

② $\underbrace{\sum_{j=0}^{N-1} f(j)\overline{g(j)}}_{\substack{\text{DEF.} \\ \langle f, g \rangle}} = \frac{1}{N} \sum_{k=0}^{N-1} \hat{f}(k)\,\overline{\hat{g}(k)}$ PARSEVAL

$$\sum_{j=0}^{N-1} |f(j)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |\hat{f}(k)|^2$$

③ Chapter 7, exercise 1

④ —"—, —"— 2

⑤ —"—, —"— 8

⑥ Show that there does not exist such a <u>function</u> $f \in L^1(\mathbb{R})$ that

$$\int_{-\infty}^{\infty} f(x)\varphi(x)\,dx = \varphi(0)$$

for all $\varphi \in C_0^\infty(\mathbb{R})$.