# FOURIER ANALYSIS
## φv. 6

① Assume that $f \in \mathcal{S}(\mathbb{R}^2)$ and that

$$\int_L f\,dl = 0$$


for every line L. Prove that $f = 0$.

Hint The Fourier transform.

② Chapter 6, problem 8a.

③ φvn. 5, ex. 1 (again!)

④ Is the function

$$\frac{1}{\cosh(x)}$$

of class $\mathcal{S}(\mathbb{R})$, the Schwartz' class.

⑤ Let $L_{t,\theta}$ denote the line

$$x\cos\theta + y\sin\theta = t.$$

For $f \in \mathcal{S}(\mathbb{R}^2)$ we define the "X-ray transform"

$$X(f)(t,\theta) = \int_{L_{t,\theta}} f = \int_{-\infty}^{\infty} f(t\cos\theta + u\sin\theta,\ t\sin\theta - u\cos\theta)\,du$$

Calculate this for $f(x,y) = e^{-\pi(x^2+y^2)}$.

⑥ Let $f(x) = X_1(x_1)X_2(x_2)\cdots X_n(x_n)$. Verify that

$$\hat{f}(\xi) = \hat{X}_1(\xi_1)\hat{X}_2(\xi_2)\cdots\hat{X}_n(\xi_n)$$

(The variables are "disjoint".)