FOURIER ANALYSIS
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① Apply the Poisson summation formula to $f(x) = e^{-a|x|}$, $a > 0$. Conclude that

$$\frac{\cosh[a(\pi - x)]}{\sinh(a\pi)} = \frac{1}{a\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{a}{a^2+n^2}\cos(nx)$$

when $0 \le x \le \pi$. (See §5.5/Ex. 19a, b.)

② $$\lim_{t\to 0+}\left\{\sqrt{t}\sum_{n=-\infty}^{\infty} e^{-n^2 t\pi}\right\} = ?$$

③ §5.6, Problem 2

④ $$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{e^{-i\omega x}\,dx}{\cosh\left(x\sqrt{\frac{\pi}{2}}\right)} = \frac{1}{\cosh\left(\omega\sqrt{\frac{\pi}{2}}\right)} \qquad (!)$$

⑤ Let $$\hat{f}(\xi) = \int_{\mathbb{R}^n} f(x)\, e^{-2\pi i\langle x,\xi\rangle}\,dx$$

If f is a radial function*), so is $\hat{f}$. Prove this by showing that $\hat{f}(\xi) \equiv \hat{f}(U\xi)$ for each unitary matrix.

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*) i.e. $f(x) = f(|x|)$, $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$