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① Study the convergence of the functions

$$f_n(x) = \frac{nx}{1+n^2x^2} \qquad (n = 1, 2, 3, \ldots)$$

Is the convergence uniform in the interval $[-1, 1]$? What about $[10^{-100}, \infty)$? A picture!

② The "functions" $\varphi_1, \varphi_2, \varphi_3, \ldots$ are orthogonal in the Hilbert space $\mathcal{H}$, i.e.,

$$\langle \varphi_k, \varphi_j \rangle = 0, \quad \text{when } j \neq k.$$

Show that $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$ is linearly independent.

③ We know that the trigonometric system is a basis in $L^2(-\pi, \pi)$. Show that the system

$$\frac{\sin(nx)}{\sqrt{\frac{\pi}{2}}} \qquad (n = 1, 2, 3, \ldots)$$

is a basis in the Hilbert space $L^2(0, \pi)$.

④ Assume that $f_n \longrightarrow f$ in $L^2(-\pi, \pi)$, i.e.,

$$\lim_{n\to\infty} \int_{-\pi}^{\pi} |f(x) - f_n(x)|^2 dx = 0.$$

Prove that $\lim_{n\to\infty} c_k(f_n) = c_k(f)$, $k = 0, \pm 1, \pm 2, \ldots$, for the Fourier coefficients.

(5) $\lim_{n\to\infty} z_n = z \implies \lim_{n\to\infty} \frac{z_1 + z_2 + \cdots + z_n}{n} = z$

- Stein – Sh., Chapter 3, Ex. 9
- —— " ——, —— " ——, Ex. 11 a
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