FOURIER ANALYSIS

TMA 4170

2013 Øv 11.

① Suppose that ψ is continuous and

i) $|\psi(x)| \leq M e^{-|x|}$

ii) $\int_{-\infty}^{\infty} \psi(x)\,dx = 0$ $\quad \left(|e^{i\theta}-1| \leq |\theta| \right)$

Show that $\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}\,d\omega < \infty.$

Hint: $|\hat{\psi}(\omega) - \hat{\psi}(0)| \leq M|\omega| \underbrace{\int_{-\infty}^{\infty} |x| e^{-|x|}\,dx}_{=2}.$

② Consider Daubechie's

$$\phi(x) = \sum_{k=0}^{3} p_k \phi(2x-k), \quad \psi(x) = \sum (-1)^k \bar{p}_{1-k} \phi(2x-k)$$

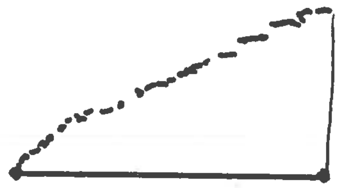
where $p_0 = \frac{1+\sqrt{3}}{4}$, $p_1 = \frac{3+\sqrt{3}}{4}$, $p_2 = \frac{3-\sqrt{3}}{4}$, $p_3 = \frac{1-\sqrt{3}}{4}$.

Calculate the moments

$$\int_{-\infty}^{\infty} \psi(x)\,dx, \quad \int_{-\infty}^{\infty} x\,\psi(x)\,dx, \quad \int_{-\infty}^{\infty} x^2 \psi(x)\,dx.$$

(3) Suppose that f is monotone*) in the interval $[0, 2\pi]$. Let

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} f(x) e^{-inx}\,dx$$



Show that

$$|c_n| \leq \frac{|f(2\pi -) - f(0+)|}{4|n|} \qquad (n = \pm 1, \pm 2, \ldots)$$

(Remark: For functions of bounded variation one can prove: $|c_n| \leq \frac{1}{4|n|} \mathrm{Var}(f)$.)

(4) Stein, p. 217

$$(-\Delta)^a f(x) = \int_{\mathbb{R}^n} (2\pi|\xi|)^{2a} \hat{f}(\xi) e^{2\pi i \langle x, \xi \rangle} d\xi$$

Chapter 6, § 7.

Problem 7a, 7c.

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*) The function is Riemann integrable, but may have denumerably many discontinuities.

This is the last exercise set!
All the exercises are included in the "pensum". EXAM: 21. V. 2013.