Öv. 3 / 2013

Peter Lindqvist

① $$2\pi D_N(x) = \sum_{n=-N}^{N} e^{inx}$$

$$F_N(x) = \frac{D_0(x) + D_1(x) + \cdots + D_N(x)}{N+1}$$

Derive the formula

$$2\pi F_N(x) = \frac{1}{N+1}\left(\frac{\sin\left(\frac{N+1}{2}x\right)}{\sin\left(\frac{x}{2}\right)}\right)^2$$

for the Fejer kernel.

---

② If the Fourier series of a continuous function f converges at a point x_0, it converges to $f(x_0)$. Proof!

---

③ Does there exist a function $f \in L^2(-\pi, \pi)$ whose Fourier coefficients are

$$c_n = \frac{1}{\sqrt{|n|}} \quad ?$$

---

④ "What happens" if $|f(x) - f(y)| \leq |x-y|^\alpha$ for all x, y and for some fixed $\alpha > 1$?

⑤ Chapter 3, exerc. 11a.

⑥ Chapter 4, exerc. 5.

⑦ Chapter 4, exerc. 9.