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FOURIERANALYSE TMA 4170 Sensurfrist 10.VI

Hjelpemidler: Bestemt kalkulator. Ett A4-ark stemplet fra Instituttet med valgfri paaskrift av studenten.

1. Let
$$c_n = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} f(x) e^{-inx}\, dx$$
be the Fourier coefficient of a function f in $L^2(-\pi, \pi)$. Prove that $c_n \to 0$, as $|n| \to \infty$.

2. Given that
$$\hat{g}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega x} g(x)\, dx = \omega^2 e^{-\frac{\omega^2}{2}},$$
find the function $g(x)$ explicitly.

3. Find one function u that satisfies the equation
$$\left(\frac{d^2}{dx^2} - 4\right) u = \delta$$
in the sense of distributions (δ is Dirac's).

4. Evaluate the integral
$$\iint e^{2\pi i \langle \mathbf{x}, \boldsymbol{\gamma} \rangle}\, dS(\boldsymbol{\gamma})$$
taken over the unit sphere $|\boldsymbol{\gamma}| = 1$ in $\mathbb{R}^3$. (Hint: first, verify that the integral depends only on $|\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$.)

5. Suppose that a continuous function ψ satisfies the conditions

$$|\psi(x)| \leq 50e^{-|x|} \quad \text{and} \quad \int_{-\infty}^{\infty} \psi(x)\, dx = 0.$$

Show that

$$\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}\, d\omega \quad < \infty.$$

(Please, pay attention to $\omega = 0$.)

6. What is the Fourier transform of the function

$$\phi(x) = \frac{\sin(\pi x)}{\pi x}\,?$$

Are the functions $\phi_k = \phi(x-k),\ k = 0, \pm 1, \pm 2, \cdots$, orthonormal in the Hilbert space $L^2(\mathbb{R})$? Prove your assertion.