FOURIER ANALYSIS

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The multiresolution analysis gave

$$\phi(x) = \sum p_k \phi(2x-k)$$

$$\psi(x) = \sum \overline{p_{1-k}} (-1)^k \phi(2x-k)$$

1. Prove $\langle \phi, \psi \rangle = 0$

2. Prove $\langle \psi_{0k}, \psi_{0l} \rangle = \delta_{kl}$

3. Let

$$\phi(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0, & |x| > 1 \end{cases}$$

Are the functions orthogonal: $\{\phi(x-k) \mid k \in \mathbb{Z}\}$?

4. If $\phi_2(x)$ satisfies the scaling relation with $p_0 = \frac{1+\sqrt{3}}{4}$, $p_1 = \frac{3+\sqrt{3}}{4}$, $p_2 = \frac{3-\sqrt{3}}{4}$, $p_4 = \frac{1-\sqrt{3}}{4}$ (otherwise $p_n = 0$), which scaling relation does $\phi_2\left(\frac{3}{2} - x\right)$ satisfy ?