FOURIER ANALYSIS

Øv. 8

① The discrete Fourier transform of f is $(0,4,0,0)$. Find $(f(0), f(1), f(2), f(3))$.

② Let $f \in \mathbb{C}^8$. We know

e = even
o = odd

$$Df_e = \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix}, \quad Df_o = \begin{pmatrix} 2 \\ -1-i \\ 0 \\ -1+i \end{pmatrix}$$

in the FFT. Find $Df(3)$ and $Df(7)$.

③ Let $$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx .$$

Express the Fourier transform of
$$f_{jk} = 2^{j/2} f(2^j x - k)$$
in terms of $\hat{f}(\omega)$.

Compact support

④ Suppose that $\phi \in C_0(\mathbb{R})$ satisfies the so-called scaling relation
$$\phi(x) = \sum_k p_k \phi(2x-k)$$
Show that
$$a = \inf\{x \mid \phi(x) \neq 0\}, \quad b = \sup\{\cdots\}$$
are integers.

(5) $$\sum_{n=-\infty}^{\infty}\left(\frac{\sin(an)}{an}\right)^2 = \int_{-\infty}^{\infty}\left(\frac{\sin(at)}{at}\right)^2 dt$$

$$0 < a < \pi$$

(6) Daubechies' Wavelet

$$\phi(x) = \sum_k p_k \phi(2x-k)$$

$$\psi(x) = \sum_k \overline{p_{1-k}}\,(-1)^k \phi(2x-k)$$

$$p_0 = \frac{1+\sqrt{3}}{4}, \quad p_1 = \frac{3+\sqrt{3}}{4}, \quad p_2 = \frac{3-\sqrt{3}}{4},$$

$$p_3 = \frac{1-\sqrt{3}}{4}, \quad p_k = 0 \text{ otherwise}$$

Verify that $\langle \phi, \psi \rangle = 0$.