FOURIER ANALYSIS

Ex. 7

2012

(1)
$$\int_0^1 \left(\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\, dx \right) dy = ?$$

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What can you say about $\iint_Q \frac{x^2-y^2}{(x^2+y^2)}\, dx\, dy$ taken over the square.

(2) Find all solutions in $\mathcal{S}'$ of

$$\frac{dT}{dx} = \delta \quad \text{(Dirac's } \delta\text{)}.$$

Does there exist any solution $T \in \mathcal{E}'$.

(3) Find the Fourier transforms of

$|x|$, $PV\left(\frac{1}{x^2}\right)$

(4) Calculate the DFT of the vector

$(0, 1, 2, \ldots, N-1)$

(5) Prove the formula for the DFT of the convolution.

(6) Compute the powers of the $N \times N$ matrix

$$\left(e^{-\frac{2i\pi}{N} jk} \right)_{j,k=0,1,2,\ldots,N-1}$$

appearing in the DFT.

(7) Is the function

$$\frac{1}{\cosh(x)}$$

of class $\mathcal{S}$ (Schwartz' class).