

① Solve

$$\frac{dT}{dx} - aT = \delta$$

in $\mathcal{D}'(\mathbb{R})$

②

Show that

$$f(x) = \begin{cases} a^{-1} \sinh(ax), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

solves

$$\frac{d^2 f}{dx^2} - a^2 f = \delta$$

in the sense of distributions.

③ Compute the Fourier transforms of $PV(\frac{1}{x})$ and $\text{sign}(x)$

④ Verify $(e^x - 1) PV(\frac{1}{x}) = \frac{e^x - 1}{x}$

⑤ $f(x) = \overline{\arctan(x)}$. To which spaces $L^1(\mathbb{R})$, $L^2(\mathbb{R})$, $\mathcal{F}(\mathbb{R})$, $\mathcal{E}'(\mathbb{R})$, $\mathcal{F}'(\mathbb{R})$ does f belong? Find \widehat{f} .

⑥ $a f(ax) \xrightarrow{\mathcal{D}'} \delta$, where $\int_{-\infty}^{\infty} f = 1$.