FOURIERANALYSE 2012

ØVN 5.

(1) Show that there does not exist such a function $f \in L^1(\mathbb{R})$ that

$$\int_{-\infty}^{\infty} f(x)\varphi(x)\,dx = \varphi(0)$$

for all $\varphi \in C_0^\infty(\mathbb{R})$.

(2) Are the following functionals distributions in $\mathcal{D}'(\mathbb{R})$?

a) $T(\varphi) = |\varphi(0)|$

b) $T(\varphi) = 1$

c) $T(\varphi) = \sum_{n=0}^{\infty} \varphi^{(n)}(0)$

d) $T(\varphi) = \int_{-\infty}^{\infty} |x|^2 \varphi(x)\,dx$

(3) Evaluate

$$\int_{-\infty}^{\infty} \left(\frac{\sin x}{x}\right)^2 dx$$

Hint: (Parseval) – Plancherel.

(4) Show that the Fourier transform of a radial function $f = f(x)$ is radial.

Remark: Radial means that it depends only on $r = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$. — Hint $f(x) = f(Ux)$ for all unitary transformations $U$ means that $f(x)$ is radial.

(5) Find $\frac{d}{dx} \log|x|$ in the sense of distributions $\mathcal{D}'(\mathbb{R})$.

(6) Verify $H' = \delta$ (Heaviside's function) carefully.