FOURIER SERIES
Exercise 4 / 2012

(1) Find $$\hat{f}(\xi) = \int_{-\infty}^{\infty} e^{-2i\pi x\xi} f(x)\,dx$$
of $f(x) = e^{-ax^2}$ by using a differential equation for $\hat{f}'(\xi)$.

(2) Prove the formula $\widehat{f * g} = \hat{f}\,\hat{g}$ when f, g belong to the class $\mathcal{S}$.

(3) Find a rapidly decaying function in $C^\infty(\mathbb{R})$ whose derivative is not rapidly decaying.

(4) Show that $f \in L^2(\mathbb{R})$

(5) $f = \chi_{[0,1]}$. $f * (f * f) = ?$

(6) $\chi_{[-a,a]} * \sin(x) = 2\sin(a)\sin(x)$

Remarks. $$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$