TMA 4170

FOURIER ANALYSIS

① If the Fourier series of a continuous periodic function f converges at a point x_0, it converges to $f(x_0)$. Proof!

② Prove that the implication

$$\lim_{n\to\infty} z_n = z \implies \lim_{n\to\infty} \frac{z_1 + z_2 + \cdots + z_n}{n} = z$$

holds for complex numbers.

③ $$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-i\omega x}}{\cosh\left(\sqrt{\frac{\pi}{2}}\, x\right)}\, dx = \frac{1}{\cosh\left(\sqrt{\frac{\pi}{2}}\, \omega\right)}$$

A contour integral!

④ Does there exist a function $f \in L^2(-\pi, \pi)$ whose Fourier coefficients are

a) $a_0 = 0; \quad a_n = b_n = 1/\sqrt{n}, \quad n = 1, 2, \dots$

b) $a_0 = 0; \quad a_n = b_n = 1/n, \quad n = 1, 2, \dots$

(5) Let $\mathcal{H}$ be a Hilbert space and assume that $v_j \longrightarrow v$ <u>weakly</u>:

$$\lim_{j\to\infty} \langle v_j, u \rangle = \langle v, u \rangle \quad \text{whenever } u \in \mathcal{H}$$

Prove that for some subsequence

$$\lim_{n\to\infty} \left\| \frac{v_{j_1} + v_{j_2} + \cdots + v_{j_n}}{n} - v \right\| = 0$$

<u>Hint</u>: You may assume that $\|v_j\| \le M$, $j = 1, 2, 3, \ldots$, which could be proved.