FOURIER

TMA 4170 Ex. 12

① If $\psi \in \mathcal{S}(\mathbb{R})$ and $\displaystyle\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

then
$$\left(\int_{-\infty}^{\infty} x^2 |\psi(x)|^2 dx\right)\left(\int_{-\infty}^{\infty} \xi^2 |\hat{\psi}(\xi)|^2 d\xi\right) \geq \frac{1}{16\pi^2}$$

according to Heisenberg's uncertainty principle. Determine when equality holds, by inspecting the proof. Here
$$\hat{\psi}(\xi) = \int_{-\infty}^{\infty} \psi(x) e^{-2\pi i x \xi} dx .$$

② Let H denote the Heaviside function and
$$v(x) = H(x) - 2H(x-1)$$

Find v' and $\hat{v}$ in the sense of distributions. Determine the supports of v, v', and $\hat{v}$.

③ Poisson's formula

$$u(r,\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1-r^2}{1-2r\cos(\theta-\varphi)+r^2}\, f(\varphi)\, d\varphi$$

solves the problem

$$\begin{cases} \Delta u = 0 & \text{in } r < 1 \\ u(1,\theta) = f(\theta) \end{cases}$$

Assume that f is continuous and periodic. Derive the formula by

- separating the variables
- using superposition
- determining the coefficients
- summing the series

<u>Remark</u>

$$u(r,\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} r^n \left(a_n \cos(n\theta) + b_n \sin(n\theta)\right)$$

$$= \sum_{n=0}^{\infty} c_n r^{|n|} e^{in\theta}$$