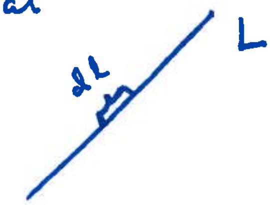
FOURIER ANALYSIS

Øv. 11 2012

① Assume that $\left|\sum c_n d_n\right| \le 1$ for all sequences d_n such that $\sum |d_n|^2 \le 1$. Show that $\sum |c_n|^2 \le 1$. The same with $\frac{1}{p} + \frac{1}{q} = 1$.

② $$\frac{1}{4\pi} \iint_{S^2} e^{-2\pi i \langle \xi, \gamma \rangle} \, dS(\gamma) = \frac{\sin(2\pi|\xi|)}{2\pi|\xi|}$$

③ Let $f \in \mathcal{S}(\mathbb{R}^2)$. Assume that



$$\int_L f \, dl = 0$$

for all lines L. Prove that $f \equiv 0$.

④ Let $L = L_{t,\theta}$ denote the line

$$x \cos\theta + y \sin\theta = t$$

in the xy-plane. For $f \in \mathcal{S}(\mathbb{R}^2)$ we define

$$X(f)(t,\theta) = \int_{L_{t,\theta}} f = \int_{-\infty}^{\infty} f(t\cos\theta + u\sin\theta,\ t\sin\theta - u\cos\theta)\, du$$

Calculate this for $f(x,y) = e^{-\pi(x^2+y^2)}$.

(5) For $F \in \mathcal{S}(\mathbb{R} \times S^1)$ we define the <u>dual</u> X-transform

$$X^*(F)(x,y) = \int F(x\cos\theta + y\sin\theta, \theta)\, d\theta$$

Check that if $f \in \mathcal{S}(\mathbb{R}^2)$, $F \in \mathcal{S}(\mathbb{R} \times S^1)$, then

$$\int_0^{2\pi}\int_{-\infty}^{\infty} X(f)(t,\theta)\overline{F(t,\theta)}\, dt\, d\theta = \iint_{\mathbb{R}^2} f(x,y)\overline{X^*(F)(x,y)}\, dx\, dy$$