FOURIER ANALYSIS

10/2012

(1) Find the support of the scaling function $\phi(x)$ in

$$\phi(x) = \sum_{k=0}^{3} p_k \phi(2x-k)$$

Then find the support of the wavelet $\psi(x)$.

(2) Prove that

$$\sum_{k=1}^{\infty} |a_k| < \infty \Rightarrow \prod_{k=1}^{\infty} (1+a_k) \text{ converges}$$

(3) Start from

$$\left(\cos^2\left(\frac{\omega}{2}\right) + \sin^2\left(\frac{\omega}{2}\right)\right)^5 = 1$$

and obtain the coefficients p_k of the scaling function, called ϕ_3.

(4) Let

$$\hat{\phi}(\omega) = \frac{1}{\sqrt{2\pi}}\left(\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}\right)\frac{1}{\sqrt{1-\frac{2}{3}\sin^2\left(\frac{\omega}{2}\right)}}$$

Prove that the translated functions $\phi(x-k)$ are orthonormal.

Remark: $$\sum_{k=-\infty}^{\infty} \frac{1}{(\omega+2k\pi)^4} = \frac{3-2\sin^2\frac{\omega}{2}}{48\sin^4\left(\frac{\omega}{2}\right)}$$

follows upon differentiating

$$\frac{1}{\sin^2\frac{\omega}{2}} = \sum_{k=-\infty}^{\infty} \frac{1}{(\omega+2k\pi)^2}.$$

twice.