FOURIER ANALYSIS Tma4170
Exercise 1 / 2012

① Let

$$\frac{a_0}{2} + \sum_{n=1}^{N} a_n \cos(nt) + b_n \sin(nt) = \sum_{n=-N}^{N} c_n e^{int}$$

Given a_n, b_n, find c_n and c_{-n}.

② Verify that $\sum_{n=-N}^{N} e^{in\theta} = \dfrac{\sin\left[\left(N+\frac{1}{2}\right)t\right]}{\sin\left(\frac{t}{2}\right)}$ †)

③ $$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sin\left[\left(N+\frac{1}{2}\right)t\right]}{\sin\left(\frac{t}{2}\right)}\,dt = ?$$

④ The system $\{\varphi_1, \varphi_2, \varphi_3, \dots\}$ is orthonormal in the Hilbert space $\mathcal{H}$.
Verify the identity

$$\left\| u - \sum_{n=1}^{N} C_n \varphi_n \right\|^2 = \|u\|^2 - \sum_{n=1}^{N} |\langle u, \varphi_n\rangle|^2 + \sum_{n=1}^{N} \|C_n - \langle u, \varphi_n\rangle\|^2 .$$

Thus the choice $C_n = \langle u, \varphi_n\rangle$ is optimal.

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†) This is the Dirichlet kernel (times 2π).