Norges teknisk–
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Contact during exam:
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EXAM IN SIF5020 LINEAR METHODS

Monday, December 11, 2000
Time 09.00-14.00

Hjelpemidler: C1 Approved calculator.
All handwritten and printed aids allowed.

Give reasons for all answers

Problem 1

a) Let E be the subset of l^∞ consisting of all sequences of numbers where at most finitely many of the numbers are different from 0. Explain why E is a normed space (with the induced norm). Is E complete?

b) Show that E is an example of a normed space that is not an inner product space.

Problem 2

a) Let $T : C[0,1] \to C[0,1]$ be given by

$$(Tx)(t) = \int_0^1 ts\; x(s)\,ds$$

Show that $\|T\| = \dfrac{1}{2}$.

b) Let $T : L^2(0,1) \to L^2(0,1)$ be given by

$$(Tx)(t) = \int_0^1 ts\, x(s)\, ds$$

Show that $\|T\| \leq \frac{1}{3}$.

c) Find a continuous function x on the interval $[0,1]$ that satisfies the integral equation

$$x(t) = 4 + \int_0^1 ts\, x(s)\, ds; \;\; 0 \leq t \leq 1.$$

Problem 3

Given real matrices A and B where A is an $m \times r$-matrix with linearly independent columns and B an $r \times n$-matrix with linearly independent rows.

Show that

$\mathcal{N}(AB) = \mathcal{N}(B)$ og $\mathcal{R}(AB) = \mathcal{R}(A)$.

Problem 4

a) Find the QR-factorization (with $r_{kk} > 0$) for

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

b) Which point in $\text{lin}\{(1,1,1),(1,2,3)\} \subset \mathbb{C}^3$ is closest to the point $(i,0,1)$?

Problem 5

a) Given $A = \frac{1}{3}\begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 1 & -1 \end{bmatrix}$.

What is e^A?

b) Given more generally $T \in \mathcal{L}(E)$ where E is a Banachspace.

Explain why the series $\displaystyle\sum_{n=0}^{\infty} \frac{T^n}{n!}$ converges in $\mathcal{L}(E)$. The limit is denoted by e^T.

Show that $0 \notin \sigma(e^T)$.

Problem 6 Is the matrix

$$A_n = \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & \cdots & \cdots & 2 \\ \vdots & \vdots & \ddots & & \\ \vdots & \vdots & & n-1 & n-1 \\ 1 & 2 & & n-1 & n \end{bmatrix}$$

positive definite?