Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2022

Exercise set 8

1 Consider the equation $f(x) = x^2 - 3$. Check that the Newton iteration

$$T(x) := x - \frac{f(x)}{f'(x)} = \frac{1}{2}\left(x + \frac{3}{x}\right)$$

maps $[\sqrt{3}, \infty)$ to itself. Then use the Banach fixed point theorem to show that

$$\lim_{n\to\infty} T^n(x) = \sqrt{3}$$

for every $x \geq \sqrt{3}$.

2 Let $G : (C[0,1], ||\cdot||_\infty) \to (C[0,1], ||\cdot||_\infty)$ be defined by

$$(Gx)(t) = \int_0^t sx(s)\, ds,\ 0 \leq t \leq 1.$$

Show that G is a contraction with zero function as the unique fixed point.

3 Apply Picard iteration to

$$x'(t) = 1 + x^2,\ \ x(0) = 0.$$

Find x_3 and the exact solution (notice that the equation is separable), and show that the terms involving $t, t^2, \cdots, t^5$ in $x_3(t)$ are the same as those of the Taylor series of the exact solution.